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AFFDL-TR-68-43
PART I

RANDOM-VIBRATION ANALYSIS SYSTEM FOR COMPLEX STRUCTURES

PART I: ENGINEERING USER'S GUIDE

*D. R. LAGERQUIST
L. D. JACOBS*

The Boeing Company

TECHNICAL REPORT AFFDL-TR-68-43, PART I

NOVEMBER 1968

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FOREWORD

The computer program reported herein was prepared by The Boeing Company for the Aero-Acoustics Branch, Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under contract AF 33(615)5155. The study demonstrates the application of finite-element matrix methods in determining the responses and fatigue lives of complex panels excited by random pressure fluctuations. This research is part of a continuing effort to establish tolerance levels and design criteria for sonic-fatigue prevention under the exploratory development program of the Air Force Systems Command. The effort was conducted under project 1471 "Aero-Acoustic Problems," task 147101 "Sonic Fatigue." Mr. D. L. Smith and Mr. M. C. Eifert of the Aero-Acoustics Branch were the task engineers.

The period covered by this effort is July 1966 through June 1968. This report is AFFDL-TR-68-43 "Random-Vibration Analysis System for Complex Structures," Part I "Engineering User's Guide" and is one of four documents prepared under contract AF 33(615)-5155. One document is Part II of this report entitled, "Computer Program Description." The other reports are AFFDL-TR-67-81 "A Finite Element Analysis of Simple Panel Response to Turbulent Boundary Layers," and AFFDL-TR-68-44 "Finite-Element Analysis of Complex Panel Response to Random Loads." The Boeing Company's document number for this report is D6-23145, Part I. Part II of this report is only available through DDC and the Air Force Flight Dynamics Laboratory (FDDA) Wright-Patterson Air Force Base, Ohio.

The research was conducted by Loyd D. Jacobs and Dr. Dennis R. Lagerquist of the Structural Dynamics Staff of The Boeing Company's Commercial Airplane Division in Renton, Washington. Acknowledgement is due Dr. J. R. Fuller and C. D. Newsom, the principal contributors to the early development of the analysis methods used.

Principal programmers for the analysis system were K. Tsurusaki and F. S. Wallace of The Boeing Company's computing department. The structural matrix program used was adapted by Dr. R. L. Sack of the Structure's Staff and R. D. Palm of the computing department. Other programming contributors were H. B. Noonchester and L. Anderson of the computing department.

This report was submitted by the authors in July 1968.

This report has been reviewed and approved.



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ABSTRACT

A user's guide is presented for a computer program developed to aid in the design of sonic-fatigue-resistant aircraft structure. The program employs matrix methods to calculate statistical measurements of response (deflection and stress) for complex structure subjected to pressure loads random in both time and space. The program is in two phases. Finite-element methods are used in the first phase to determine structural characteristics such as flexibility, natural frequencies, and modes of vibration. In the second phase, a cross-power spectral density loading function, is generated and combined with structural characteristics to compute response. Either cross-power spectral density or joint statistical moments, including second spectral moments useful in fatigue analysis, can be computed for response. The loading function models a decayed progressive wave typical of laboratory noise sources. Different loading functions can be supplied by the user, because the program is constructed in modular form. The program was written for the IBM 7094 computer primarily in FORTRAN IV language with a MAP language matrix manipulation module.

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NOMENCLATURE

A	beam cross-sectional area
$[A]$	flexibility matrix
$[A]$	diagonal matrix of elemental areas on structure
A_{XY}, A_{XZ}	beam shear areas in Y and Z directions, respectively
$[C]$	damping matrix
$[C_F(\omega)]$	force co-PSD matrix
$[C_p(\omega)]$	pressure co-PSD matrix
c_t	pressure-wave trace velocity
c_x, c_y	phase velocities in x and y directions, respectively
$[C_\delta(\omega)]$	deflection co-PSD matrix
$[C_\sigma(\omega)]$	stress co-PSD matrix
$[D]$	dynamic matrix
$[D]$	diagonal matrix in real part of admittance matrix
d	pressure-wave decay constant
$[E]$	diagonal matrix in imaginary part of admittance matrix
$\{F\}$	column matrix of applied forces
F_X, F_Y, F_Z	element forces in X , Y , and Z directions, respectively
g	structural damping coefficient
$[H(i\omega)]$	admittance matrix of complex-frequency-response functions
$[H_j(i\omega)]$	diagonal matrix of admittance terms
$[I]$	identity matrix
I_X, I_Y, I_Z	moments of inertia about X , Y , and Z directions, respectively
I_s	moment of inertia for sandwich plate

J	beam torsion constant
$[J(\omega)]$	real part of admittance matrix
K	number of modal cross products in analysis
$[K]$	stiffness matrix
$[L(\omega)]$	imaginary part of admittance matrix
$[M]$	mass matrix
M_j	generalized mass
M_X, M_Y, M_Z	element bending moments about X , Y , and Z axes, respectively
M_{XY}	plate element twisting moment
m	number of normal modes
N_0	expected number of zero crossings per second of response time history
N_{XY}	plate element in-plane shear
n	number of kinematic degrees of freedom
$[Q_F(\omega)]$	force quad-PSD matrix
$[Q_p(\omega)]$	pressure quad-PSD matrix
Q_X, Q_Y	plate element out-of-plane shears on Y-Z and X-Z planes, respectively
$[Q_\delta(\omega)]$	deflection quad-PSD matrix
$[Q_\sigma(\omega)]$	stress quad-PSD matrix
$[S]$	matrix of stress-deflection relationships
t	time (sec)
t_o	plate thickness
t_s	effective shear thickness of plate
t_X, t_Y	effective plate thickness added by stiffeners for membrane forces in X and Y directions, respectively

NOMENCLATURE (concluded)

X, Y, Z	local Cartesian coordinates for structural element
x, y, z	Cartesian coordinates
α, β, γ	direction angles of free-space wave-front velocity vector
$\{\delta\}$	column matrix of deflections
$[\overline{\delta\delta}]$	deflection covariance matrix
$\delta_x, \delta_y, \delta_z$	deflections in x , y , and z directions, respectively
ζ	critical damping ratio
η	separation distance in y direction
θ	angle of wave-front propagation
$\theta_x, \theta_y, \theta_z$	rotations about x , y , and z axes, respectively
λ, μ	proportionality factors for stiffness and mass-proportional damping, respectively
λ_j	eigenvalue
ν	Poisson's ratio
ξ	separation distance in x direction
$\{\sigma\}$	column matrix of stress
$[\overline{\sigma\sigma}]$	stress covariance matrix
$\Phi(\omega)$	pressure PSD $(\text{psi})^2(\text{sec})$
$[\Phi_F(i\omega)]$	force cross-PSD matrix
$[\Phi_\delta(i\omega)]$	deflection cross-PSD matrix
$[\Phi_\sigma(i\omega)]$	stress cross-PSD matrix
$[\phi]$	matrix of eigenvectors
$\{\phi(j)\}$	eigenvector
ω	angular frequency (rad/sec)
$[]^T$	transpose of matrix
$()^*$	complex conjugate
$(\ddot{})$	second derivative with respect to time

I

INTRODUCTION

For many years there has been a need for better methods of sonic fatigue analysis for aircraft structure. Current methods commonly rely on available test data that are not always adaptable to new designs. A design tool is needed by which sonic fatigue performance can be established as a design parameter.

RANVIB (Random Vibration Analysis System for Complex Structures) is a computer program developed to aid in sonic fatigue analyses. The program employs matrix methods to calculate statistical measurements of response (deflection and stress) for complex structure subjected to random noise fields.

The basis of the analysis method is a combination of matrix structural analysis and random-process theory. Load-displacement relationships are determined by finite-element analyses, and vibrational properties are determined by computing eigenvalues and eigenvectors. These characteristics are used to form structural admittance functions. Force loads such as noise pressure waves are based on using cross-power spectral density (cross PSD) on a mathematical model. Response is related to the loading by admittance functions.

The program is written primarily in FORTRAN IV language for the IBM 7094 computer. However, a machine language module is used for matrix manipulations.

The computer program report is in two parts:

1. Part I—Engineering User's Guide
2. Part II—Computer Program Description

This first volume is a guide for an engineer's use of RANVIB. The method of analysis used in the program is described in section II. Section III describes the use of the program. Sample problems for application to the program are presented in section IV.

A report oriented to programmer use is presented separately in Part II, (reference 1). Development of the theory of this program and its application to specific problems are presented in AFFDL-TR-68-44 (reference 2). An earlier study that used portions of this program is reported in reference 3.

II ANALYSIS METHOD

The method of analysis combines matrix structural analysis techniques and random process theory to determine structural response to random pressure loads. The computer program is modular with each module performing a specific function in the analysis.

As shown in figure 1, the program is organized into two separate phases. The first phase contains the structural generation and the eigenvalue and eigenvector solution modules. Excitation generation and response solution are in the second phase. Information from phase I is transferred to phase II by means of tape.

1. SCOPE OF APPLICATIONS

This program will help the analyst design structure that is resistant to sonic fatigue. This is done by providing information on the random stress response distribution on small sections of complex structure responding to random pressure loads. These stress distributions can be used to determine where regions of intense stress occur and, hence, provide stress information necessary for fatigue life prediction. Knowledge of the stress distribution allows the analyst to redesign to relieve these stresses and, thereby, provide increased fatigue life. Used in parallel with tests, this program can provide design charts on complex structural configurations and minimize the need for large numbers of costly tests.

The program will accommodate large complex structural systems that exhibit linear elastic behavior and are represented by beam and plate elements. Features of the program include a general three-dimensional beam element, quadrilateral and triangular plate elements (isotropic or orthotropic) with both in-plane and out-of-plane stiffnesses, element in-plane preload capability, and elastic support capability. Inertia characteristics are determined by masses lumped at discrete points.

Applied loading is a force that fluctuates randomly in both space and time. It is assumed that the loading is a stationary, ergodic, random process. The cross PSD of forces applied at structural node points in the direction of nodal deflections is used to describe the excitation.

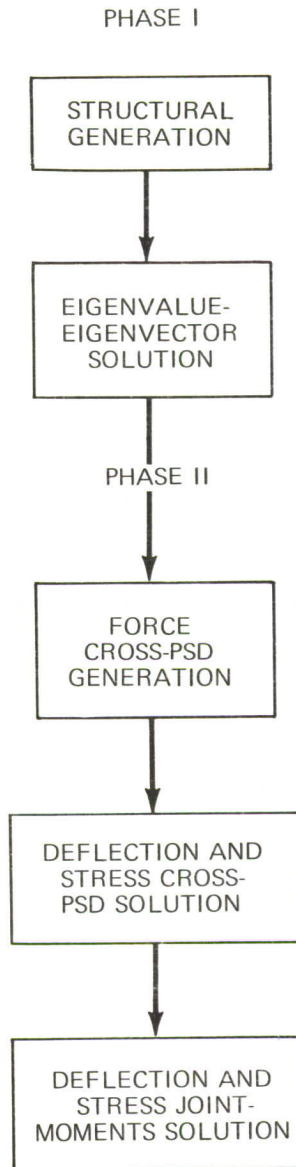


Figure 1. Computer Program Organization

The excitation subroutine used in this report describes a mathematical model of a laboratory noise source. This subroutine can be replaced with one that simulates any other excitation source. The present form of this subroutine restricts use of this program to rectangular panels.

Calculated responses include deflections and stresses. These stresses are actually internal forces in the structural elements, and element geometry must be used to determine true stress (force per unit area). Either response covariance or cross PSD can be determined. In addition, stress second spectral moments can be used to find the expected number of zero crossings in stress time history for use in predicting fatigue life. Calculations for expected number of zero crossings are valid only when response-level distributions are Gaussian.

Analysis options for various forms of system damping are available in the program. A general analysis option allows damping of a general viscous form. Simplified analyses can be performed if certain assumptions are made about damping.

2. PHASE I—STRUCTURAL AND VIBRATIONAL ANALYSIS

Structural stiffness, flexibility, and stress-deflection matrices are generated in the first phase of the program. The flexibility matrix and mass data are then used to determine the natural frequencies and modes of vibration. The results of the first phase can be examined to determine the adequacy of the structural representation before proceeding to the response-solution phase.

a. Structural Matrix Generation

(1) General Information

The program module that generates stiffness, flexibility, and stress matrices was adapted from a larger matrix structural analysis program (reference 4). This program is capable of analyzing large complex structural systems by the stiffness or displacement method.

The structure is described in Cartesian coordinates by a set of control points (nodes) connected by plates and beams. Each node is assigned 6 degrees of freedom: rotations about each of the coordinate axes and displacements in

the directions of the coordinate axes. The constraint conditions for each degree of freedom are:

- (a) Free
- (b) Fixed (zero displacement)
- (c) Sprung (a real or imaginary spring attached to a ground point)

Either quadrilateral or triangular plates may be used for structural representation, but best stress results are obtained for rectangular elements. Beams may be straight or curved and may have uniform or nonuniform section properties.

The normal assumptions in structural analysis are made, i.e.:

- (a) The material is perfectly elastic
- (b) The deflections are sufficiently small compared to the size of the structure that secondary deflections, caused by interaction between the applied forces and primary deflections, are negligible.

This program is limited to the evaluation of structures that can be adequately described by no more than 2,000 nodes. The total number of plates and/or beams is limited only by this restriction. Further, the nodes are grouped into partitions. The maximum number of rows of partitions is 200, and the maximum number of nodes per partition is 10. The total number of partitions in the stiffness matrix is limited to 800.

(2) Fundamental Matrix Equations

The concepts of the matrix structural analysis are contained in texts such as reference 5. A few of the fundamentals are presented herein.

Most structural engineers are familiar with deflection influence coefficients a that relate the deflections δ at one point on a structure to external forces F applied at that and other points. For any point on a structure, an equation may be written in the form

$$\delta_i = a_{i1} F_1 + a_{i2} F_2 + \dots + a_{ii} F_i \dots + a_{in} F_n \quad (1)$$

If all n equations for this structure are written, the resulting set of equations may be collected and expressed in matrix form as

$$\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdot & \cdot & \cdot & a_{1n} \\ a_{21} & a_{22} & \cdot & \cdot & \cdot & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \cdot & \cdot & \cdot & a_{nn} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{Bmatrix} \quad (2)$$

or

$$\{\delta\} = [A] \{F\} \quad (3)$$

This matrix of deflection influence coefficients is called the flexibility matrix.

A less familiar concept is that of the force influence coefficients K that relate the external force at one point on a structure to deflections at that and other points. For any point on a structure, an equation may be written in the form

$$F_i = K_{i1} \delta_1 + K_{i2} \delta_2 + \cdot \cdot \cdot + K_{ii} \delta_i + \cdot \cdot \cdot + K_{in} \delta_n \quad (4)$$

The collection of these force equations that is comparable to the flexibility matrix is

$$\begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & \cdot & \cdot & \cdot & K_{1n} \\ K_{21} & K_{22} & \cdot & \cdot & \cdot & K_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ K_{n1} & K_{n2} & \cdot & \cdot & \cdot & K_{nn} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \end{Bmatrix} \quad (5)$$

or

$$\{F\} = [K] \{\delta\} \quad (6)$$

The matrix of force influence coefficients is conventionally called a stiffness matrix, and the individual terms in the matrix are called the stiffness coefficients. The stiffness matrix is the inverse of the flexibility matrix.

(3) Matrix Generation

Generation of the stiffness matrix is accomplished by generating individual element matrices and merging them into a complete structural stiffness matrix. Generation of element stiffnesses is discussed briefly in appendix I. The total stiffness matrix is modified to include boundary conditions of the supported nodes.

Not all nodal displacement freedoms are retained in the dynamic-response analysis performed by the RANVIB program. Thus, the structural matrices must be compatible with the retained freedoms. The so-called "reduced" stiffness matrix has the apparent effect of selecting a smaller set of nodal freedoms in the matrix. Reduced freedoms, which are assumed to be unloaded by either applied or inertia forces, do not appear explicitly but are taken into account. The complete stiffness matrix is rearranged and partitioned so that the retained freedoms are separated from the unloaded freedoms to be reduced. Then equation (6) can be written as

$$\begin{Bmatrix} F_1 \\ 0 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \quad (7)$$

The reduced displacements are given by

$$\{\delta_2\} = -[K_{22}]^{-1} [K_{21}] \{\delta_1\} \quad (8)$$

Therefore,

$$\{F_1\} = [[K_{11}] - [K_{12}] [K_{22}]^{-1} [K_{21}]] \{\delta_1\} \quad (9)$$

The reduced stiffness matrix is

$$[K_r] = [K_{11}] - [K_{12}] [K_{22}]^{-1} [K_{21}] \quad (10)$$

The corresponding reduced flexibility matrix is equal to the inverse of $[K_r]$.

When nodal displacements are obtained, they can be used to obtain internal forces (stresses) in the individual members. The stress deflection relationships can be written in matrix form as

$$\{\sigma\} = [S] \{\delta\} \quad (11)$$

where $\{\sigma\}$ is a column of member stresses and $[S]$ is the stress-deflection transformation (stress) matrix. The stress matrix derived in the program is a reduced matrix determined in the same manner as $[K_r]$. Equation (11) can be written as

$$\{\sigma\} = [S_1 \mid S_2] \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \quad (12)$$

Substitution of equation (8) into equation (12) results in

$$\{\sigma\} = \left[[S_1] - [S_2] [K_{22}]^{-1} [K_{21}] \right] \{\delta_1\} \quad (13)$$

Therefore, the reduced stress matrix is given by

$$[S_r] = [S_1] - [S_2] [K_{22}]^{-1} [K_{21}] \quad (14)$$

For the remainder of this report, the r subscript is deleted and the structural matrices used refer to reduced matrices.

Some problems require an analysis that will include the effects of large axial (or in-plane) loads. Obviously, these can cause a significant change in stiffness of the structure. This effect can be included by using the so-called geometric stiffness matrix. Hence, a complete stiffness matrix can be formed so that

$$[K] = [K_0] + [K_1] \quad (15)$$

where: $[K_0]$ = the usual stiffness matrix

$[K_1]$ = the geometric stiffness matrix

Now equation (6) can be written by substituting equation (15). All subsequent equations, partitions, etc., are valid after this substitution. For a more-complete discussion of this concept, see reference 6. A description of the $[K_1]$ matrices is contained in appendix I.

b. Vibration Analysis

The vibration analysis module of the program determines natural frequencies and modes of vibration for the system. These vibrational characteristics are determined from the undamped, unforced equation of motion:

$$[K] \{\delta(t)\} + [M] \{\ddot{\delta}(t)\} = 0 \quad (16)$$

where t denotes time. The mass matrix $[M]$ contains inertia coefficients corresponding to the retained freedoms of the idealized system.

Solutions of equation (16) are assumed in the form

$$\{\delta(t)\} = \{\phi^{(j)}\} \exp \{i(\omega_j t + \psi)\} \quad (17)$$

where: $\{\phi^{(j)}\}$ = a vector describing a deflected shape

ω_j = a circular frequency

ψ = a phase angle

Substitution of equation (17) into equation (16) results in the classical eigenvalue problem

$$|[D] - \lambda[I]| = 0 \quad (18)$$

where $[I]$ is the identity matrix.

The dynamic matrix $[D]$ is given by

$$[D] = [K]^{-1} [M]$$

and

$$\lambda = \frac{1}{\omega^2}$$

Each eigenvalue λ_j determines a natural circular frequency ω_j . Associated with each eigenvalue is an eigenvector $\{\phi^{(j)}\}$ that represents the mode shape for that frequency.

The eigenvalues of the dynamic matrix are solved by the QR method (reference 7) in this program. This method determines all eigenvalues by a series of transformations of the dynamic matrix. In general, only about the lowest 25 percent of the frequencies and modes are physically realistic because of the idealized representation of the system.

The flexibility matrix is read from tape in this module and the mass matrix is read from cards. To provide a diagonal mass matrix, the inertia characteristics are represented by lumped point masses. The maximum matrix size is of 100th order, and up to 25 modes can be determined.

3. PHASE II—RANDOM LOADS AND RESPONSES

In phase II, results from phase I and information obtained from the excitation module are used to determine responses in the various solution modules. Phase II can be repeated using different solution options without repeating phase I.

a. Random Pressure Loads

(1) General Considerations

The RANVIB analysis system was constructed for use on structural systems responding to a class of fluctuating pressures called sonic loads. The system can also be used when there are other types of external forces. The only requirements are that the applied forces be

- (a) ergodic, stationary, random processes, and
- (b) independent of structural response level (aerodynamic forces excluded).

Matrices describing applied forces are generated by a random-loading module (called RANLOD) in the phase II program. This loading module describes convective, random, fluctuating pressures by use of a mathematical model based on decayed progressive waves. This form of loading frequently occurs in sonic test laboratories where reverberation on reflection is small. The output of RANLOD is force cross-PSD matrices. RANLOD can be replaced by user-supplied modules that describe other types of loadings meeting the general requirements stated above. The loading module for attached turbulent boundary layers described in reference 3 is an example of the use of another type of random-loading module. The following section describes the loading module for the decayed progressive wave; it serves as an example of what must be considered if the user wishes to provide his own RANLOD.

(2) Decayed Progressive Wave

Sound pressure on structural panel sections located at great distances from any type of random sound source can be approximated by decaying plane, progressive waves. An example of a progressive wave is shown in figure 2. The wave front will generally intersect with the structural surface at an oblique angle so that trace wave fronts propagate across the panel forming what is

called progressive waves. Normal incidence (plane waves parallel to the surface of a flat panel) and grazing incidence (plane waves perpendicular to the surface of a flat panel) are special cases.

Force cross-PSD matrices $[\Phi_F(i\omega)]$ are generated and used in the phase II solution program. These matrices are Hermitian. They can be described as the sum of a real matrix and an imaginary matrix; these matrices are force co-power $[C_F(\omega)]$ and quad-power $[Q_F(\omega)]$ spectral density (co-PSD and quad-PSD) matrices:

$$[\Phi_F(i\omega)] = [C_F(\omega)] + i[Q_F(\omega)] \quad (19)$$

The $i\omega$ denotes that cross PSD is a complex function of frequency ω .

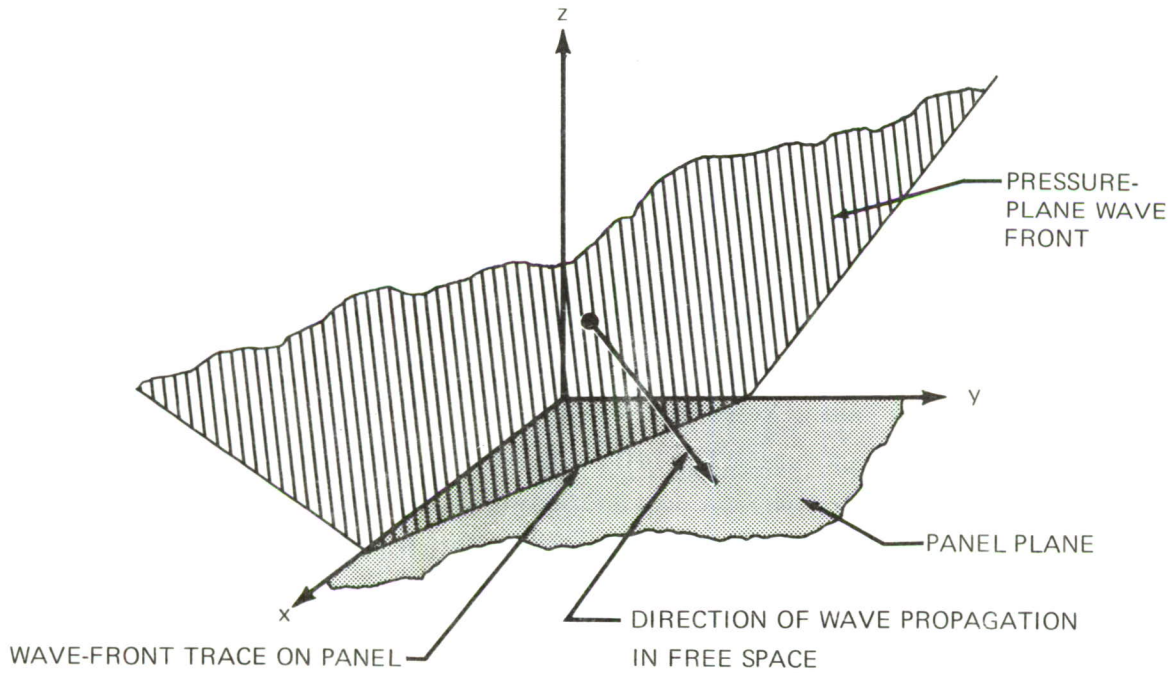


Figure 2. Pressure Wave and Structural Panel

The sonic type of distributed pressure loads are related to force by the following equations:

$$[C_F(\omega)] = [A] [C_p(\omega)] [A] \quad (20)$$

$$[Q_F(\omega)] = [A] [Q_p(\omega)] [A] \quad (21)$$

where: $[C_p(\omega)]$ = pressure co-PSD matrix

$[Q_p(\omega)]$ = pressure quad-PSD matrix

$[A]$ = a diagonal matrix of areas associated with each retained node

When the fluctuating pressure load is modeled by a decayed progressive wave, the terms of the pressure loading matrices are (reference 2) as follows:

$$C_{p_{ij}}(\omega) = \Phi(\omega) \exp [-d (\xi_{ij}^2 + \eta_{ij}^2)^{1/2}] \cos \left[\frac{\omega}{c_t} (\xi_{ij} \cos \theta + \eta_{ij} \sin \theta) \right] \quad (22)$$

$$Q_{p_{ij}}(\omega) = \Phi(\omega) \exp [-d (\xi_{ij}^2 + \eta_{ij}^2)^{1/2}] \sin \left[\frac{\omega}{c_t} (\xi_{ij} \cos \theta + \eta_{ij} \sin \theta) \right] \quad (23)$$

where: $\Phi(\omega)$ = pressure PSD

d = decay constant

ξ_{ij} = x-direction separation distance between node points i and j

η_{ij} = y-direction separation distance between node points i and j

θ = propagation angle of the wave front over the panel

Angle θ and trace velocity c_t are shown in figure 3.

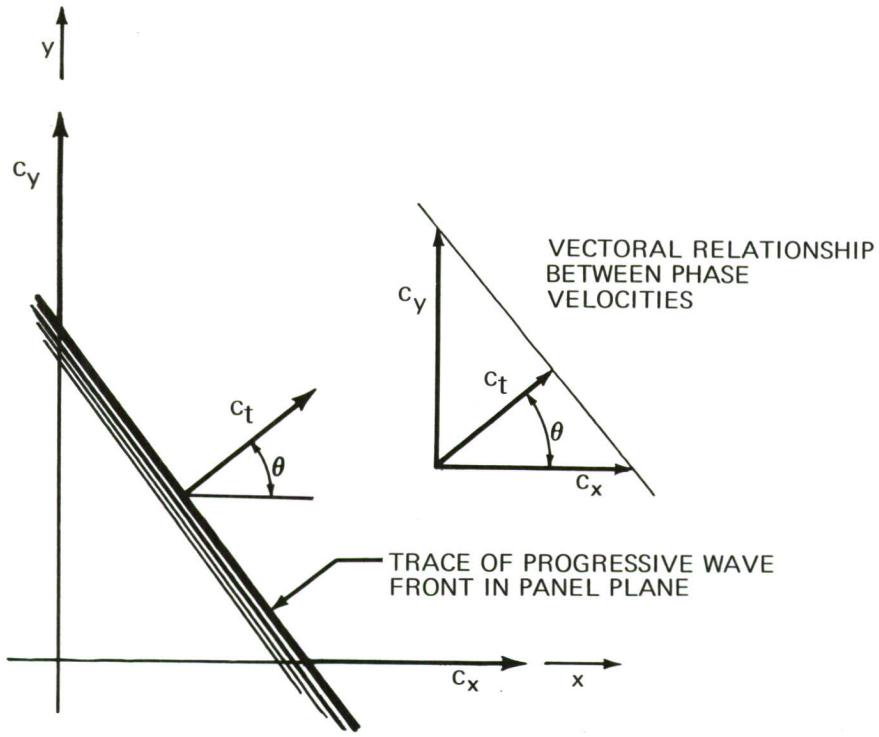


Figure 3. Pressure-Wave Propagation on Panel Surface

The angle θ defines the direction that the trace of the pressure wave fronts propagate over the panel surface. This angle is defined by

$$\theta = \sin^{-1} \left\{ \frac{1}{\left[1 + \left(\frac{c_y}{c_x} \right)^2 \right]^{1/2}} \right\} \quad (24)$$

where c_x and c_y are the respective phase velocities of the pressure wave in the x and y directions of the panel. Wave-front phase velocities in the plane of the panel are related as follows:

$$c_t = c_x \cos \theta \quad (25)$$

$$c_t = c_y \sin \theta \quad (26)$$

When angle θ approaches zero, a more accurate definition of c_t results when using the cosine equation. One method of calculating c_x and c_y is from knowledge of wave front transit times along the panel edges τ_x and τ_y and of the edge lengths a and b. Thus, $c_x = a/\tau_x$ and $c_y = b/\tau_y$.

The loading function has two general forms, which depend upon how pressure wave fronts sweep across the panel. The general forms are as follows:

- (a) Normal-incidence waves
- (b) Progressive waves

Normal-incidence waves occur when wave fronts are parallel to the panel surface. In this case, the trace velocity on the panel approaches infinity, and the trigonometric functions in equations (22) and (23) have constant values:

$$\cos \left[\frac{\omega}{c_t} (\xi_{ij} \cos \theta + \eta_{ij} \sin \theta) \right] = 1 \quad (27)$$

$$\sin \left[\frac{\omega}{c_t} (\xi_{ij} \cos \theta + \eta_{ij} \sin \theta) \right] = 0 \quad (28)$$

The quad PSD is everywhere zero, and co-PSD simplifies to the product of pressure PSD and a spatial decay term.

A progressive wave occurs whenever trace velocity c_t has finite value. There are three types of progressive waves:

- (a) General ($\theta \neq \frac{n\pi}{2}$, $c_t < c_x$, $c_t < c_y$)
- (b) Wave progressive in the x direction ($\theta = n\pi$, $c_t = c_x$)
- (c) Wave progressive in the y direction ($\theta = \frac{(2n-1)\pi}{2}$, $c_t = c_y$)

where n is any integer. When the pressure wave is progressive in the x or y direction, respectively, trigonometric terms in equations (22) and (23), respectively, reduce to

$$\begin{aligned} & \cos \left[\frac{\omega}{c_x} \xi_{ij} \right] \\ & \sin \left[\frac{\omega}{c_x} \xi_{ij} \right] \end{aligned} \quad \text{(wave progressive in the } x \text{ direction)}$$

$$\begin{aligned} & \cos \left[\frac{\omega}{c_y} \eta_{ij} \right] \\ & \sin \left[\frac{\omega}{c_y} \eta_{ij} \right] \end{aligned} \quad \text{(wave progressive in the } y \text{ direction)}$$

The type of loading function generated by the program is controlled by option and parameter inputs. Selection of normal incidence or progressive wave forms is controlled by an option control card. In addition, the type of progressive-wave loading function that is generated is controlled by the input phase-velocity parameters. When the wave is progressive in the direction of one of the coordinate axes, phase velocity in the perpendicular direction is infinite. A zero is used to indicate this infinite phase velocity in one direction. The presence or absence of a zero-coordinate trace velocity then controls selection of the type of progressive wave loading.

The pressure spectra characteristics are determined by $\Phi(\omega)$. These values are card inputs. A pressure PSD must be the input for each frequency ω used in the solution computations in phase II. These frequencies are either card inputs or automatic inputs from the phase I analysis of natural frequencies. The source of frequency data depends upon whether or not normal modes are used in the solution analysis and upon whether a response PSD or joint moment is calculated. Refer to section II 3.b. for a discussion of response solutions.

Separation distances are calculated from coordinates. The separation distances between node points i and j are

$$\begin{aligned}\xi_{ij} &= x_j - x_i \\ \eta_{ij} &= y_j - y_i\end{aligned}\tag{29}$$

The locations of node points are determined by the idealization used in phase I to simulate the structure. Most structures analyzed for response to sound are panel sections that can be idealized by locating all node points on the set of intersections formed by groups of lines parallel to the x and y axes (figure 4). RANLOD requires that this structural idealization be used here so that separation and areas for pairs of node points are calculated automatically; hence the present form of the loading module restricts program use to rectangular panels. The advantage of this is that the total set of coordinates need not be used. The only inputs that are necessary are the sets of line-to-origin distances. The area A for node k then is calculated automatically using

$$A_k = \frac{1}{4} (x_{i+1} - x_{i-1}) (y_{j+1} - y_{j-1})\tag{30}$$

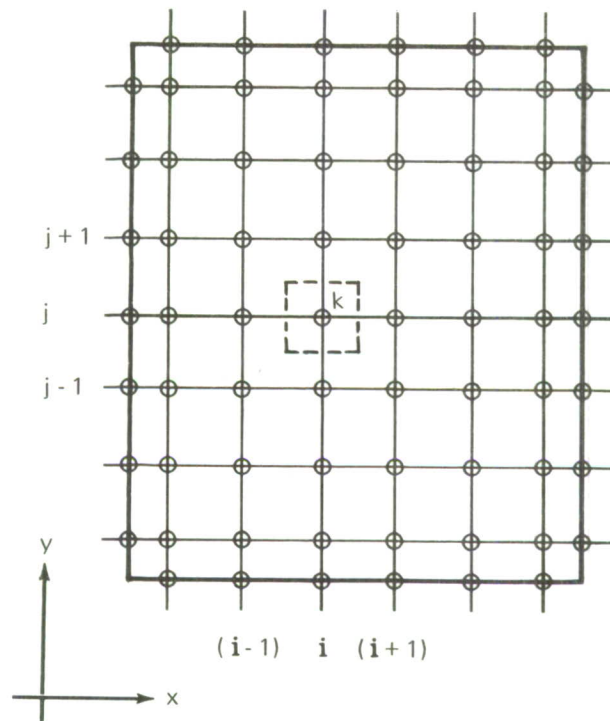


Figure 4. Structural Nodes on Rectangular Grid

This set of areas is grouped to form diagonal area matrices $[A]$ used to premultiply and postmultiply pressure relationships as shown in equation (21).

Each node point of a structure has six potential freedoms of motion, but the freedoms of most node points are not retained in a structural analysis. Certain node points, such as those representing a boundary of a clamped plate, are totally constrained and have no freedoms. Other nodes may have some or no constraints but are not directly loaded by an applied or inertial force. The freedoms of these nodes need not be retained in the structural analysis, because the stiffness matrix of retained freedoms includes their effects.

The freedoms that are retained are of node points that are directly loaded. The applied force from sonic or small-scale fluctuating pressures acts normal to the surface of an aircraft skin; the pressure-loaded finite elements of these skin panels have only the translational freedoms normal to the panel surface (vertical). Thus, the pressure loads that must be included in a force cross-PSD matrix are normal forces corresponding to the node points with retained vertical translation freedoms. The program requires that inertial loads correspond to these same retained vertical freedoms. This is because the automated separation generation feature precludes generating a cross-PSD loading function with zero load on any retained freedom.

The Hermitian properties of $[\Phi_F(i\omega)]$ can be used to simplify the form of the co- and quad-PSD matrices. The Hermitian property is

$$[\Phi_F(i\omega)] = [\Phi_F^*(i\omega)]^T$$

where the asterisk denotes the complex conjugate, and $[\]^T$ denotes the transpose of the matrix. Terms of the matrices have the following properties:

$$C_{F_{ii}}(\omega_k) = C_{F_{jj}}(\omega_k) = \Phi_F(\omega_k) \quad (31)$$

$$C_{F_{ij}}(\omega_k) = C_{F_{ji}}(\omega_k) \quad (32)$$

$$Q_{F_{ii}}(\omega_k) = Q_{F_{jj}}(\omega_k) = 0 \quad (33)$$

$$Q_{F_{ij}}(\omega_k) = -Q_{F_{ji}}(\omega_k) \quad (34)$$

The force co- and quad-PSD matrices are constructed by ordering terms to match the order in the structural stiffness matrix developed in phase I. The force co-PSD matrix is

$$[C_F(\omega_k)] = \begin{bmatrix} \Phi(\omega_k) & C_{12}(\omega_k) & C_{13}(\omega_k) & \dots & C_{1n}(\omega_k) \\ C_{12}(\omega_k) & \Phi(\omega_k) & C_{23}(\omega_k) & \dots & C_{2n}(\omega_k) \\ C_{13}(\omega_k) & C_{23}(\omega_k) & \Phi(\omega_k) & \dots & C_{3n}(\omega_k) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_{1n}(\omega_k) & C_{2n}(\omega_k) & C_{3n}(\omega_k) & \dots & \Phi(\omega_k) \end{bmatrix} \quad (35)$$

The force quad-PSD matrix is

$$[Q_F(\omega_k)] = \begin{bmatrix} 0 & Q_{12}(\omega_k) & Q_{13}(\omega_k) & \dots & Q_{1n}(\omega_k) \\ -Q_{12}(\omega_k) & 0 & Q_{23}(\omega_k) & \dots & Q_{2n}(\omega_k) \\ -Q_{13}(\omega_k) & -Q_{23}(\omega_k) & 0 & \dots & Q_{3n}(\omega_k) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -Q_{1n}(\omega_k) & -Q_{2n}(\omega_k) & -Q_{3n}(\omega_k) & \dots & 0 \end{bmatrix} \quad (36)$$

where subscript n is the number of retained freedoms. For simplicity, the F subscript is dropped in the terms of the matrices.

Diagonal terms of the co-PSD matrix are the collection of PSD's at all node points. The pressure on all node points is assumed to be approximately constant, so that all diagonal terms are equal. If N frequencies are to be used in the response solution, there will be N co-PSD and N quad-PSD matrices.

When phase velocities c_x and c_y are neither known nor readily obtainable, it may be necessary to calculate c_t and θ directly from knowledge of the direction of propagation of the free-space wave fronts. This can be done by using direction cosines to define a normal to the pressure wave front. Angle γ is generally known as the angle of incidence. These direction cosines are shown in figure 5. The panel is in the xy plane. The relations are

$$\cos\theta = \frac{\cos\alpha}{\sin\gamma} \quad (37)$$

$$\left. \begin{aligned} c &= c_x \cos\alpha \\ c &= c_y \cos\beta \end{aligned} \right\} \quad (38)$$

$$c = c_t \sin\gamma \quad (39)$$

where: c = acoustic velocity of free-space wave front
 α, β, γ = angles used to form the direction cosines of vector \vec{c}
 θ = angle c_t makes with the x axis

Phase velocities c_x and c_y can be calculated by using equation (26), and the input of c_x and c_y can then be used in the program to calculate c_t and θ .

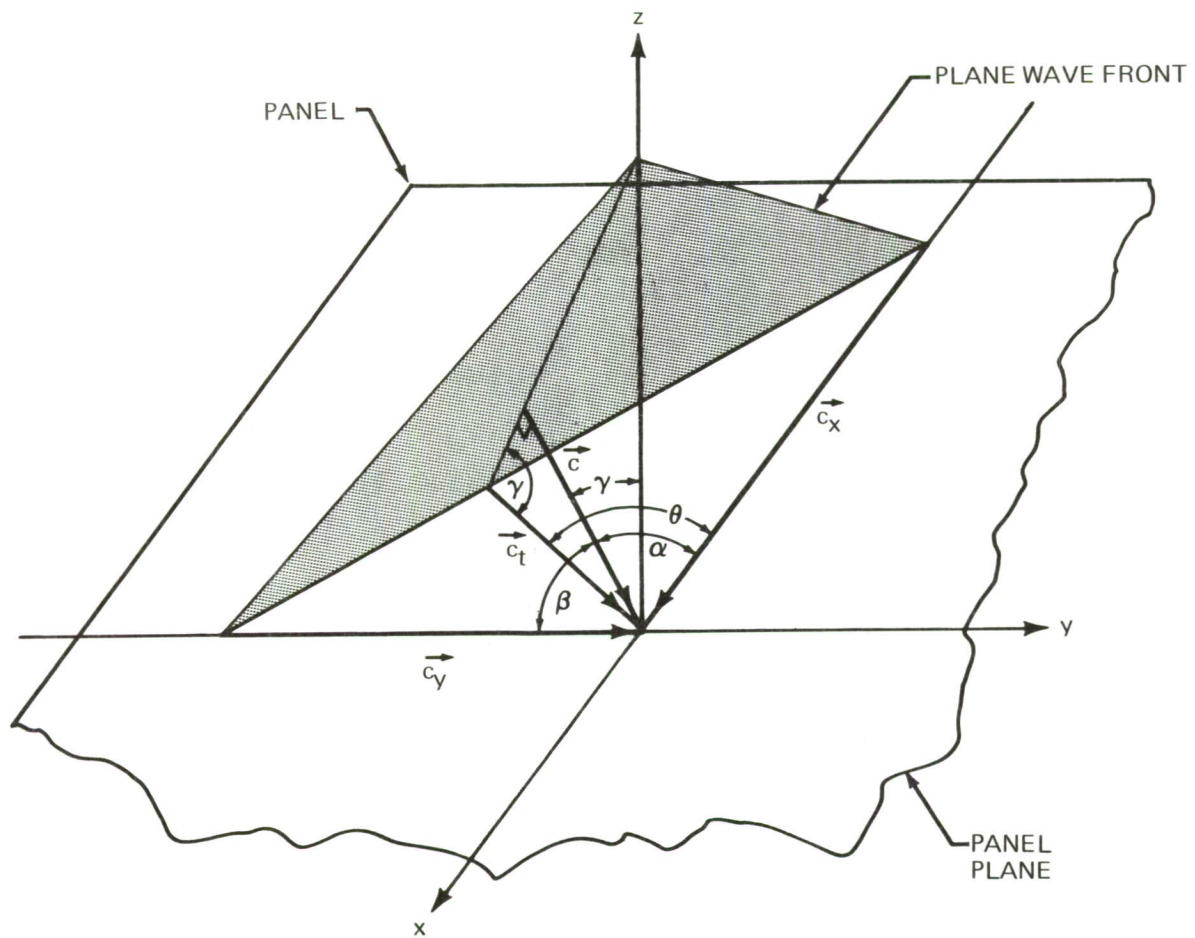


Figure 5. Pressure-Wave Group and Phase Velocities

b. Random Response

(1) Cross-Power Spectral Density (Cross PSD)

Response time history cannot be determined because pressure loads on structure vary randomly with time. However, since it is assumed that the loads are ergodic, stationary, random processes that can be described statistically, certain statistical information about the random responses can be obtained from the loading description and structural characteristics.

One quantity useful for describing random responses is cross PSD. Reference 2 shows that deflection cross PSD $[\Phi_{\delta}(i\omega)]$ is related to cross PSD for forces acting at structural node points $[\Phi_F(i\omega)]$ by the expression

$$[\Phi_{\delta}(i\omega)] = [H^*(i\omega)] [\Phi_F(i\omega)] [H(i\omega)]^T \quad (40)$$

where the diagonal elements of $[\Phi_{\delta}(i\omega)]$ are the spectral densities of the deflections, and structural characteristics are described by the admittance matrix of complex frequency-response functions $[H(i\omega)]$. The admittance matrix (reference 2) is given by

$$[H(i\omega)] = [-\omega^2 [M] + i\omega [C] + [K]]^{-1} \quad (41)$$

Matrix $[C]$ is the matrix of viscous damping coefficients. Stress response, or internal element force, is related to deflection response by stress-deflection relationships. The cross-PSD matrix of element stresses $[\Phi_{\sigma}(i\omega)]$ is given as

$$[\Phi_{\sigma}(i\omega)] = [S] [\Phi_{\delta}(i\omega)] [S]^T \quad (42)$$

Element S_{ij} of matrix $[S]$ is the i^{th} elemental stress induced by a unit displacement at j only.

(2) Joint Moments

Other information on random responses is obtained by integrating response cross PSD. The integrals of cross PSD over frequency with powers of frequency as weighting functions are known as joint spectral moments. When the power of frequency is zero, the integrals are known simply as joint moments.

The integral of a deflection PSD gives the variance or mean-square value of that deflection:

$$\overline{\delta^2} = \int_0^\infty \Phi_\delta(\omega) d\omega \quad (43)$$

The rms value of the deflection is obtained from the square root of its variance. Similarly, an average value can be obtained for the products of the deflections at points q and r from the corresponding cross-PSD term:

$$\overline{\delta_q \delta_r} = \int_0^\infty \Phi_{\delta_{qr}}(i\omega) d\omega \quad (44)$$

All such terms can be considered simultaneously and written as a matrix integration:

$$[\overline{\delta\delta}] = \int_0^\infty [\Phi_\delta(i\omega)] d\omega \quad (45)$$

The elements of the joint-moment matrix in equation (45) are space cross-correlations or covariances of the deflection responses. The diagonal elements are the mean-square deflection values.

The stress covariance matrix can be obtained by substituting stresses for deflections in equation (45). The result is

$$\begin{aligned} [\overline{\sigma\sigma}] &= \int_0^\infty [\Phi_\sigma(i\omega)] d\omega \\ &= [S] [\overline{\delta\delta}] [S]^T \end{aligned} \quad (46)$$

Square roots of the diagonal terms of the stress covariance matrix give rms stresses.

Information useful in a fatigue analysis can be obtained from second spectral moments. For a stationary Gaussian random process, the expected number of

zero crossings N_0 of a stress time history is related to the second- and zero-order moments by the expression

$$N_0 = \frac{1}{\pi} \left[\frac{\int_0^\infty \omega^2 \Phi_\sigma(\omega) d\omega}{\int_0^\infty \Phi_\sigma(\omega) d\omega} \right]^{1/2} \quad (47)$$

The stress second moments are obtained from the matrix equation

$$\int_0^\infty \omega^2 [\Phi_\sigma(i\omega)] d\omega = [S] \int_0^\infty \omega^2 [\Phi_\delta(i\omega)] d\omega [S]^T \quad (48)$$

The number of zero crossings for a particular stress component is obtained by substituting corresponding diagonal elements of the matrices given by equations (46) and (48) into equation (47).

(3) Analysis Options

The program has three basic options for calculating the responses described in paragraphs II 2.b. (1) and II 2.b. (2).

(a) General Viscous Damping (Option 1)

Option 1 is a general analysis in which it is assumed that damping is viscous and the fluctuating pressure has an arbitrary frequency distribution.

In solving the cross PSD, the matrix of damping coefficients is read from cards, and the admittance matrix is formed for each frequency specified. Since a matrix interpretive module is used in the matrix calculations, complex quantities are expressed by their real and imaginary parts. Therefore, real numbers only are used in matrix operations. Equation (41) then becomes

$$[H(i\omega)] = [J(\omega)] - i[L(\omega)] \quad (49)$$

where

$$\begin{aligned} [L(\omega)] &= [[\omega C] + [K - \omega^2 M] [\omega C]^{-1} [K - \omega^2 M]]^{-1} \\ [J(\omega)] &= [\omega C]^{-1} [K - \omega^2 M] [L(\omega)] \end{aligned}$$

The deflection cross PSD for each of the frequencies is found in terms of co-PSD $[C_\delta]$ and quad PSD $[Q_\delta]$ from equation (40) using the symmetry properties of the admittance matrix

$$[\Phi_\delta(i\omega)] = [C_\delta(\omega)] + i[Q_\delta(\omega)] \quad (50)$$

where

$$\begin{aligned} [C_\delta] &= [J] [C_F] [J] + [L] [C_F] [L] + [J] [Q_F] [L] - [L] [Q_F] [J] \\ [Q_\delta] &= [L] [C_F] [J] - [J] [C_F] [L] + [J] [Q_F] [J] + [L] [Q_F] [L] \end{aligned}$$

If stress cross PSD is desired, equation (42) is used to obtain

$$[\Phi_\sigma(i\omega)] = [C_\sigma(\omega)] + i[Q_\sigma(\omega)] \quad (51)$$

where

$$\begin{aligned} [C_\sigma(\omega)] &= [S] [C_\delta(\omega)] [S]^T \\ [Q_\sigma(\omega)] &= [S] [Q_\delta(\omega)] [S]^T \end{aligned}$$

If joint moments are to be computed, the cross-PSD matrices are numerically integrated using the trapezoidal rule. The deflection covariance matrix is found approximately by the expression

$$\begin{aligned} [\overline{\delta\delta}] &\approx \frac{\omega_2 - \omega_1}{2} [\Phi_\delta(i\omega_1)] \\ &+ \sum_{j=2}^{N-1} \frac{\omega_{j+1} - \omega_{j-1}}{2} [\Phi_\delta(i\omega_j)] + \frac{\omega_N - \omega_{N-1}}{2} [\Phi_\delta(i\omega_N)] \end{aligned} \quad (52)$$

where N is the number of frequencies for which cross PSD has been determined.

To obtain reasonable accuracy for covariance, cross PSD must be defined at an adequate number of frequencies over the significant response frequency range. Second-deflection spectral moments can be determined in a similar manner by multiplying each term in the summation on the right-hand side of equation (52) by the square of the corresponding frequency. Stress covariance and second-moment matrices can then be determined by using equations (46) and (48), respectively.

(b) Normal Modes (Option 2)

Solving for deflection cross PSD using option 1 generally involves a considerable number of matrix inversions and multiplications and, thus, may be quite time consuming. If certain assumptions are made about the form of damping present in the system, the motion equations can be uncoupled when normal mode shapes are used to describe displacements. The resulting simplified analysis is option 2, which is used in the solution module of the program.

Damping is assumed to be proportional viscous or structural, or a combination of the two forms. Viscous damping is proportional to stiffness and/or inertia. The damping matrix is given by

$$[C] = \mu[M] + \lambda[K] \quad (53)$$

where μ and λ are proportionality factors. For structural damping, the matrix $[C]$ in equation (41) is replaced by

$$\omega[C] = g[K] \quad (54)$$

where g is the structural damping coefficient. It is shown in reference 2 that these forms of damping allow the equations of motion to be uncoupled so that the matrix inversion for the admittance matrix in equation (41) reduces to the inversion of a diagonal matrix:

$$[H(i\omega)] = [\phi] [H_j(i\omega)] [\phi]^T \quad (55)$$

where

$$H_j(i\omega) = \frac{1}{M_j (\omega_j^2 - \omega^2 + i 2\zeta_j \omega \omega_j)}$$

The term ω_j is the j^{th} natural circular frequency of the system, each column of $[\phi]$ is a normal mode shape vector $\{\phi^{(j)}\}$, and the modal damping factor ζ_j is the fraction of critical damping of the j^{th} mode. The term M_j is the j^{th} generalized mass defined by

$$M_j = \{\phi^{(j)}\}^T [M] \{\phi^{(j)}\} \quad (56)$$

The modal damping factor ζ_j is related to the damping parameters by the expression

$$2\zeta_j = \frac{\mu}{\omega_j} + \lambda \omega_j \quad (57)$$

A similar approximate relationship for small structural damping is

$$g = 2\zeta_j \quad (58)$$

The admittance matrix in equation (55) can be expressed in terms of its real and imaginary parts:

$$[H_j(i\omega)] = [D] - i [E] \quad (59)$$

where the elements of the diagonal matrices are

$$D_j = \frac{1}{M_j} \frac{\omega_j^2 - \omega^2}{(\omega_j^2 - \omega^2)^2 + (2\zeta_j \omega_j \omega)^2} \quad (60)$$

$$E_j = \frac{1}{M_j} \frac{2\zeta_j \omega_j \omega}{(\omega_j^2 - \omega^2)^2 + (2\zeta_j \omega_j \omega)^2} \quad (61)$$

Substituting equation (55) into equation (40) and simplifying, the expression for deflection cross PSD becomes

$$\begin{aligned} [\Phi_\delta(i\omega)] &= [C_\delta(\omega)] + i[Q_\delta(\omega)] = \\ &\sum_{j=1}^m \sum_{k=1}^m \left\{ (D_j D_k + E_j E_k) [C_F^{(jk)}(\omega)] \right. \\ &\quad + D_j E_k ([Q_F^{(jk)}(\omega)] + [Q_F^{(jk)}(\omega)]^T) \\ &\quad + i [D_k E_j ([C_F^{(jk)}(\omega)] - [C_F^{(jk)}(\omega)]^T) \\ &\quad \left. + (D_j D_k + E_j E_k) [Q_F^{(jk)}(\omega)] \right\} \quad (62) \end{aligned}$$

where

$$[C_F^{(jk)}(\omega)] = \{\phi^{(j)}\} \{\phi^{(j)}\}^T [C_F(\omega)] \{\phi^{(k)}\} \{\phi^{(k)}\}^T$$

$$[Q_F^{(jk)}(\omega)] = \{\phi^{(j)}\} \{\phi^{(j)}\}^T [Q_F(\omega)] \{\phi^{(k)}\} \{\phi^{(k)}\}^T$$

The double summation in equation (62) is over m normal modes, where m is equal to or less than the number of degrees of freedom used to represent the system. For most problems, reasonable accuracy is obtained when m is substantially less than the number of degrees of freedom.

For lightly damped systems, the direct modal products ($j = k$) and the cross products for which numbers j and k are nearly equal are the most significant contributors to the solution in equation (62). Thus, some of the modal cross-product terms can be omitted in the calculations with no significant change in results but with considerable savings in computing time. Specified to the program in this option can be parameter K , which identifies how many cross-product terms are to be computed for each of the modes. Then only terms for which $|j - k| \leq K$ are considered in equation (62).

To compute deflection covariance in this option, it is assumed that response occurs primarily in the regions of natural frequencies (narrow-band response) and that excitation varies slowly near these frequencies (broad-band excitation). In this case, the integral in equation (45) can be evaluated by treating the excitation cross PSD as constant compared to the admittance terms. Evaluation of the "constant" excitation matrices midway between the appropriate natural frequencies and integration of equation (62) results in

$$\begin{aligned} [\overline{\delta\delta}] = & \sum_{j=1}^m \sum_{k=1}^m \left\{ [C_F^{(jk)}(\omega_{jk})] \int_0^\infty (D_j D_k + E_j E_k) d\omega \right. \\ & + ([Q_F^{(jk)}(\omega_{jk})] + [Q_F^{(jk)}(\omega_{jk})]^T) \int_0^\infty D_j E_k d\omega \\ & + i ([C_F^{(jk)}(\omega_{jk})] - [C_F^{(jk)}(\omega_{jk})]^T) \int_0^\infty D_k E_j d\omega \\ & \left. + [Q_F^{(jk)}(\omega_{jk})] \int_0^\infty (D_j D_k + E_j E_k) d\omega \right\} \end{aligned} \quad (63)$$

where

$$\omega_{jk} = \frac{\omega_j + \omega_k}{2}$$

The integrals in equation (63) have been evaluated for small damping in reference 8, and results are summarized in appendix II. As in the cross-PSD calculation, only a specified number of the cross-product terms of equation (63) are computed.

Second spectral moments of deflection can be obtained in a manner similar to the calculation of the covariance matrix. The only difference being that the integrands of the integrals in equation (63) are multiplied by ω^2 . These new integrals were also evaluated in reference 8 and are summarized in appendix II.

Stress cross-PSD and joint moment matrices are computed in this option in the same manner as in the general analysis (option 1). The corresponding deflection-response matrices are premultiplied by the stress matrix $[S]$ and postmultiplied by $[S]^T$.

(c) Normal Modes With Cross Terms Omitted (Option 3)

Option 3 of the response-solution module is a further simplification of the normal-mode analysis. In this option, all modal cross-product terms are omitted in the calculation of response cross PSD and joint moments. The accuracy of this analysis is usually sufficient for very lightly damped systems, and computation time is significantly less than that required for the other analysis options.

Disregarding cross-product terms in equation (62) and utilizing the Hermitian properties of the excitation cross-PSD matrices, the equation for deflection cross PSD reduces to a single summation:

$$[C_\delta(\omega)] \approx \sum_{j=1}^m (D_j^2 + E_j^2) \{\phi^{(j)}\} \{\phi^{(j)}\}^T [C_F(\omega)] \{\phi^{(j)}\} \{\phi^{(j)}\}^T \quad (64)$$

Response quad PSD cannot be obtained in this simplified analysis, because the resulting imaginary component is null.

Deflection covariance matrix evaluation is also simplified when cross-product terms are ignored. Omitting cross-product terms from equation (63) and evaluating the remaining admittance integral results in deflection covariance being expressed as

$$[\overline{\delta_q \delta_r}] \approx \sum_{j=1}^m \{\phi^{(j)}\} \{\phi^{(j)}\}^T [C_F(\omega_j)] \{\phi^{(j)}\} \{\phi^{(j)}\}^T \frac{\pi}{4M_j^2 \omega_j^3 \zeta_j} \quad (65)$$

The resulting imaginary component of the covariance matrix is also null in this analysis.

An expression similar to equation (65) results when evaluating deflection second spectral moments. The term ω_j^3 in the denominator of the right-hand side of equation (65) is replaced by ω_j for each term of the summation.

Stress-response matrices are determined from the corresponding deflection matrices in the same way as that in the other analysis options.

III USE OF PROGRAM

1. COMPUTER CONFIGURATION REQUIREMENTS

The RANVIB programs were written in FORTRAN IV and MAP to be run under the 7094 Mod II IBSYS operating system. By proper input/output definitions, it has been executed successfully under a coupled 7040/7094 system.

2. TIMING AND OUTPUT ESTIMATES

a. Phase I

Execution time for phase I of the program depends on many parameters, including the number of node points, total number of kinematic degrees of freedom, number of retained freedoms, and number of structural elements. The exact relationship between these factors and computing time is not known. However, based on previous problems, a rough time estimate can be obtained from the number of nodes. For problems in which the number of nodes is less than about 150, the execution time in minutes is equal to about one fourth the number of nodes. When stress-deflection matrices are not computed, the time is about 20-percent less.

For larger problems, computing time is greater than estimated by the above linear relationship. However, not enough information is available to establish a guide for estimating time for these problems.

The amount of printed output also varies with problem size. A guide for estimating the number of output lines, excluding program loading maps, has been established. Structural input data are printed as they are read, creating about one line of output for each node, plate element, and beam element in the idealization. Further output depends on the number of retained freedoms n and the number of computed mode shapes m . An approximate expression for the number of additional output lines is

$$50 + n \left[1 + 3 \left(\frac{n}{7} \right) + \left(\frac{m}{6} \right) \right] + m \left[2 + \left(\frac{m}{9} \right) \right] \quad (66)$$

The numbers in parentheses should be rounded up to the next whole number. Optional diagnostic printing of elemental structural matrices can create much additional output.

b. Phase II

Execution time for phase II depends upon analysis and solution options as well as problem size. Most of the information available is for the option 3 analysis. For deflection and stress joint-moment solutions, an approximate value for execution time is 0.008 (m n) minute. The time is about 30-percent less when second spectral moments are not computed.

Execution time (minutes) required for an option 3 cross-PSD analysis for N frequencies is approximately

$$0.004 \text{ nmN} \quad (67)$$

In an option 2 analysis, cross-modal response terms are added to option 3 responses to obtain the total results. Thus, the increase in execution time over a corresponding option 3 computing time depends upon the number of cross products included in the analysis. For K cross products for each mode, option 2 execution time is roughly $(1 + 2K)$ times the value for an option 3 analysis.

Little information is available on option 1 execution times, because only small problems have been used with this option. The time for a cross-PSD solution for a small problem was about the same as an option 2 analysis. Computation time is expected to be quite substantial for large problems, however, since inversions and multiplications of n by n matrices are performed in the analysis. Also, if joint moments are to be calculated, deflection cross PSD must be computed at a sufficient number of frequencies to obtain reasonable accuracy in the numerical integration process.

The amount of printed output for phase II also depends upon the analysis and solution options. For an option 3 analysis for deflection cross PSD at N frequencies, the number of lines of output data is approximately

$$100 + N[3 + m + n (\frac{n}{7})] \quad (68)$$

If any generated force cross-PSD matrices are printed, about $n(\frac{n}{6})$ additional lines are printed for each matrix. Stress response adds 15 lines of output for each beam element and 9 lines for each plate element for each of the N frequencies specified.

An option 3 analysis for deflection covariance and second spectral moments results in about $80 + 2n(\frac{n}{7})$ lines of output. Stress response matrices add about 30 lines for each beam element and 18 lines for each plate element. Optional printing of force cross-PSD matrices again adds $n(\frac{n}{6})$ lines for each matrix.

Both real and imaginary parts of the loading and response matrices are printed for an option 2 analysis. As a result, an option 2 analysis creates about twice as much output as a corresponding option 3 analysis.

The number of lines of output data for an option 1 deflection cross-PSD solution at N frequencies is approximately

$$80 + 3N + n [2N + 1] (\frac{n}{7}) \quad (69)$$

Optional printing of force cross PSD adds $2n(\frac{n}{6})$ lines for each matrix. If stress cross PSD is determined, the output is increased by 30 lines and 18 lines for each beam and plate element, respectively. For deflection covariance and the second-spectral-moment solution, the amount of output is the same as that for a deflection cross-PSD solution plus $4n(\frac{n}{7})$ lines. Stress covariance and second-spectral-moment responses each add the same amount of output as that of stress cross PSD.

3. INPUT DATA

a. Control

A diagram of the deck setup for phase I and phase II is shown in figure 6. There are two sets of control cards followed by the problem data cards.

The first set of control data is related to the computer installation. Cards such as job number, timing, and print estimates are included in this set. Also, identification of tapes for the program and input/output must be specified.

Control cards for the program subroutines that are on tape are contained in the second set of control data. Listings of these control cards are presented in appendix III, pages 80 and 83, for phase I and phase II, respectively.

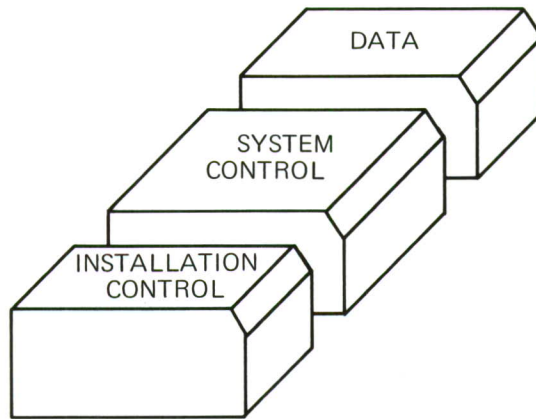


Figure 6. Deck Setup

b. Input

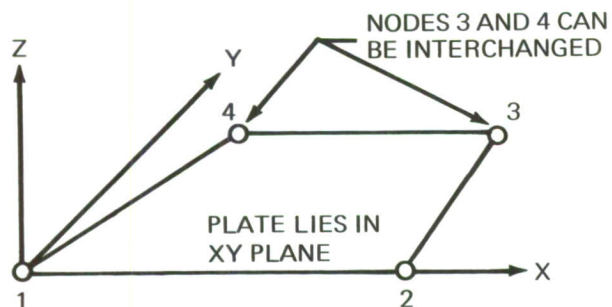
(1) Phase I Data

Input to phase I of the program is in the form of data cards. These data describe structural information necessary to generate structural matrices and to determine vibrational characteristics. The input is organized into the following:

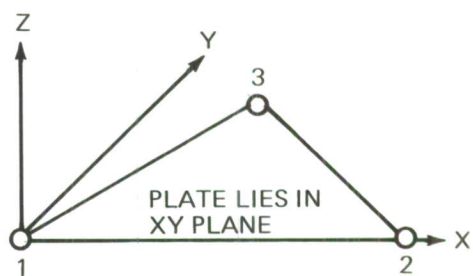
- (a) Control—structure size, partition sizes, and output options
- (b) Nodal data—nodal coordinates and constraint conditions (fixed, free, etc.)
- (c) Plate data—node numbers and properties (moments of inertia, areas, etc.)
- (d) Beam data—node numbers and section properties
- (e) Eigenvalue data—matrix size, number of modes, and mass matrix

NOTE: All input units of length and force are assumed to be inches and pounds, respectively.

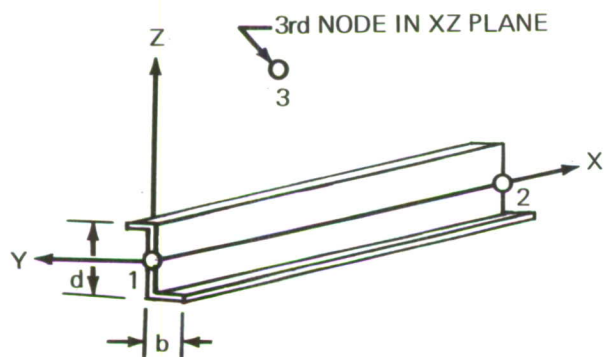
There are two types of coordinate systems. The location of nodes defining a structural assembly are defined by a structural coordinate system designated by Cartesian coordinates with lower-case letters x , y , and z . Each element has its own local coordinate system defined by cartesian coordinates designated by capital letters X , Y , and Z . These local coordinate systems are shown in figure 7 and are defined as follows:



(a) Quadrilateral Plate



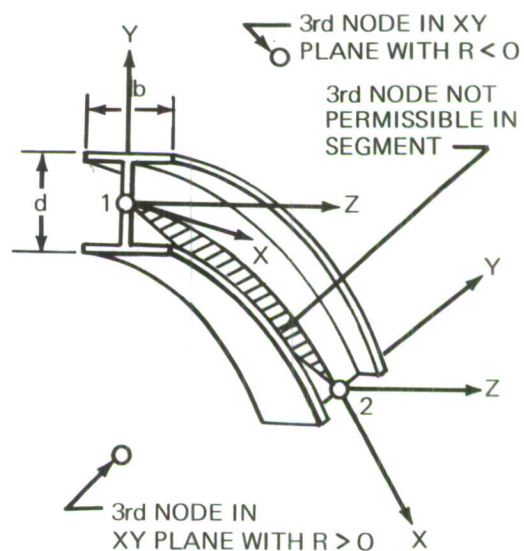
(b) Triangular Plate



SHEAR AREAS: $A_{XZ} = t_{web} \cdot d$

$A_{XY} = t_{fl} \cdot b \cdot 2 \cdot \frac{2}{3}$

(c) Straight Beam



SHEAR AREAS: $A_{XZ} = t_{fl} \cdot b \cdot 2 \cdot \frac{2}{3}$

$A_{XY} = t_{web} \cdot d$

(d) Curved Beam

Figure 7. Local Coordinate Systems

- (a) Plates—The local X axis originates at node 1 and is oriented in the direction of node 2. The local Y axis originates at node 1 and is oriented perpendicular to the local X axis in the plane of nodes 1, 2, and 3 in such a way that node 3 has a positive Y value. The local Z axis is established by the right-hand rule.
- (b) Straight beams—The local X axis originates at node 1 and is oriented in the direction of node 2. The local Z axis originates at node 1 and is oriented perpendicular to the local X axis in the plane of nodes 1, 2, and 3 in such a way that node 3 has a positive Z value. The local Y axis is established by the right-hand rule. If the 3rd node is zero, the local Z axis is assumed to originate at node 1 and be parallel to the structural z axis.
- (c) Curved beams—The local X axis originates at node 1 and is oriented in the direction of node 2 tangential to the curve. The local Y axis originates at node 1 and is oriented perpendicular to the local X axis in a radially outward direction in the plane of curvature of the beam. Node 3 defines the plane of curvature and must, therefore, be in that plane and can be located in the half plane (created by a straight line connecting nodes 1 and 2) not containing the beam. For this case, enter the radius of the curve as positive. Node 3 can be located in the half plane containing the beam but not in the segment between the beam and the line connecting nodes 1 and 2. For this case, enter the radius of the curve as negative. The local Z axis is established by the right-hand rule. The local coordinate system at node 2 is established by moving the coordinate system from node 1 to node 2 along the curved centerline of the beam and maintaining the local Y axis in a radially outward direction.

The following shows the input-data preparation procedures for phase I:

(a) Control

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
1	<u>Option control card</u>	01-10	<u>Program Option</u>	I 10
			0—Generate structural matrices	
			1—Eigenvalue solution only for previous phase I run. Previous output tape must be mounted on logical unit 10. Skip to section (e) data.	
		11-20	<u>Stress Option</u>	I 10
			0—No stresses	
			1—Plate stresses	
			2—Beam stresses	
			3—Plate and beam stresses	
		21-30	<u>Stiffness-Matrix Print Option</u>	I 10
			0—No print	
			1—Print	
		31-40	<u>Elemental Plate-Stress Matrices Print Option</u>	I 10
			0—No print	
			1—Print	
		41-50	<u>Elemental Beam-Stress Matrices Print Option</u>	I 10
			0—No print	
			1—Print	

NOTE: The elemental stress-matrix print option creates much additional output and is normally not used.

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
2	<u>Title card</u>	01-78	Title	13A6
3	<u>Matrix generation option</u>	01-24	Use one of the following two alphanumeric codes: DEFLECTIONS ONLY—Generate stiffness and flexibility matrices only. DEFLECTIONS AND STRESSES—Generate stress matrices also.	4A6
4	<u>Structure size</u>	01-04	Number of beams	I 4
		05-08	Number of plates	I 4
		09-12	Number of nodes (2,000 maximum)	I 4
		13-16	Blank	4 X
		17-20	Number of partitions (200 maximum); if 0, the program automatically generates 1 partition per 10 nodes, which is the upper limit.	I 4
5	<u>Partition sizes (Omit this card if columns 17 through 20 of card 4 are zero.)</u>	01-04, 05-08, etc.	Last node of each consecutive partition	18I4

(b) Nodal Data (one card set per node)

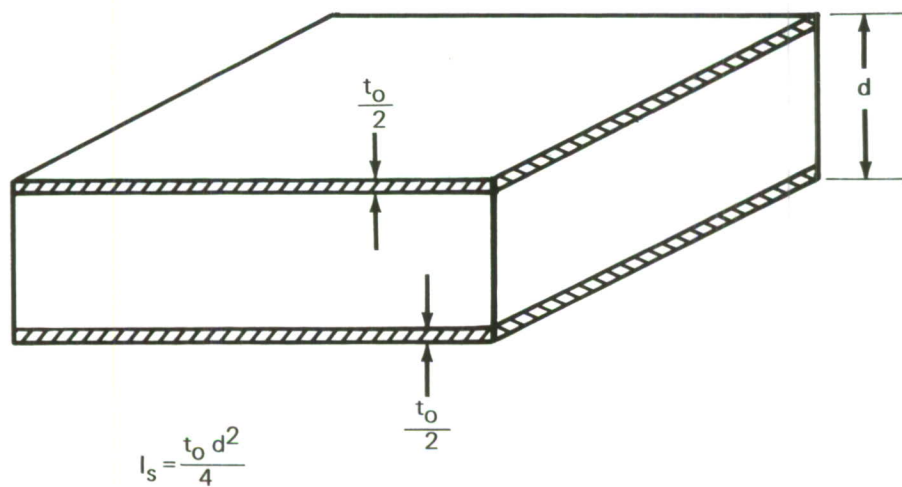
Card No.	Card title	Column	Description	Format
1	Node description	01-04	Node number (must start with 1 and continue in sequence) It is recommended that the nodes of a grid be numbered consecutively in the direction of fewer nodes. See figure 11 for phase II compatibility.	I 4
		05-10	Leave blank	6 X
		11-16	Constraint condition for freedoms($\theta_x, \theta_y, \theta_z, \delta_x, \delta_y, \delta_z$)	6 I 1
			0—Free	
			1—Fixed	
			3—Attached to a spring	
		17-24	Leave blank	8 X
		25-36	x coordinate (inches)	E12. 4
		37-48	y coordinate	E12. 4
		49-60	z coordinate	E12. 4
		65-70	Retained freedoms ($\theta_x, \theta_y, \theta_z, \delta_x, \delta_y, \delta_z$)	6 I 1
			0—Reduce	
			1—Retain	
			(Only freedoms with constraint condition 0 or 3 can be retained.) (100 maximum retained freedoms)	
2	Spring constants (If constraint conditions $\neq 3$, omit this card.)	01-12	Spring constant (in. -lb/rad) for rotation θ_x	E12. 4

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
2 (Cont.)	<u>Spring constants</u>			
		13-24	Spring constant (in. -lb/rad) for rotation θ_y	E12. 4
		25-36	Spring constant (in. -lb/rad) for rotation θ_z	E12. 4
		37-48	Spring constant (lb/in.) for deflection in x direction	E12. 4
		49-60	Spring constant (lb/in.) for deflection in y direction	E12. 4
		61-72	Spring constant (lb/in.) for deflection in z direction	E12. 4

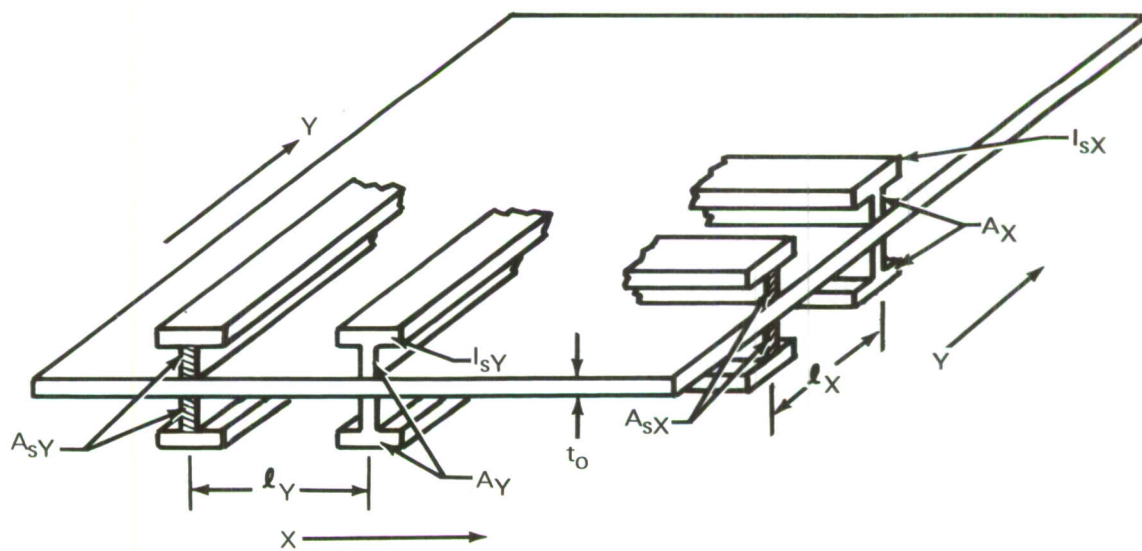
(c) Plate Data

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
1	<u>Overall plate card (Only one card for entire structure)</u>	01-12	Young's Modulus (psi)	E12.4
		13-24	Poisson's ratio	E12.4
2	<u>Plate card</u>	01-04	Plate number (must start with 1 and be in sequence)	I 4
		05-08	Node 1	I 4
		09-12	Node 2	I 4
		13-16	Node 3	I 4
		17-20	Node 4 (Leave blank if triangular plate. Nodes 3 and 4 may be specified in any order.)	I 4
		21-24	Input option	I 4
			0—Repeat previous plate properties	
			1—Uniform solid plate	
			2—Stiffened or sandwich plate	
		25-28	Output option	I 4
			0—No diagnostic print for this plate element	
			1—Coordinate-transformation, stiffness, and stress matrices are printed for this plate element.	
		29-32	Blank	4 X
		33-34	Geometric stiffness matrix option for this plate element	I 2
			0—None used	
			1—Being used; read card 2A.	

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
2 (Cont.)	<u>Plate card</u>			
		35-36	Blank	2 X
		37-48	t_o smeared area (sq in./in.) capable of carrying in-plane shear (actual thickness of the plate)	E12.4
		49-60	Young's modulus (psi) if different from overall value; otherwise leave blank	E12.4
		61-72	Poisson's ratio if different from overall value; otherwise blank	E12.4
2A	<u>Geometric stiffness matrix input</u>			
	(Omit this card if card 2, column 34, is either blank or 0.)	01-10	Direct stress in the local X direction on the plate (psi) (tension positive)	E10.4
		11-20	Direct stress in the local Y direction on the plate (psi) (tension positive)	E10.4
		21-30	Shear stress on the plate (psi)	E10.4
3	<u>Stiffened or sandwich (figure 8) (Omit this card unless card 2, column 24 is 2.)</u>			
		01-12	t_x smeared stiffener area (sq in./in.) carrying membrane forces in local X direction; $t_x = \frac{A_x}{l_x}$	E12.4
		13-24	I_y smeared moment of inertia (in. ⁴ /in.) of stiffeners about local Y axis; $I_y = \frac{I_{sx}}{l_x}$	E12.4
		25-36	t_y smeared stiffener area (sq in./in.) carrying membrane forces in the local y direction;	E12.4
		37-48	I_x smeared moment of inertia (in. ⁴ /in.) of stiffeners about local X axis; $I_x = \frac{I_{sy}}{l_y}$	E12.4



(a) Sandwich Plate



$$t_Y = \frac{A_Y}{l_Y}$$

$$t_X = \frac{A_X}{l_X}$$

$$t_s = \frac{1}{2} \left[\frac{A_{sX}}{l_X} + \frac{A_{sY}}{l_Y} \right]$$

$$I_X = \frac{I_{sY}}{l_Y}$$

$$I_Y = \frac{I_{sX}}{l_X}$$

(b) Stiffened Plate

Figure 8. Sandwich and Stiffened Plates

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
3 (Cont.)	Stiffened or sandwich	49-60	t_s smeared web area (sq in./in.) of the stiffeners	E12.4
		61-72	I_s smeared moment of inertia (in. ⁴ /in.) of that part of the sandwich plate effective in resisting in-plane loads	E12.4
NOTE: Further description of the program interpretation of the plate input data is contained in appendix I.				

Repeat cards 2 through 3 for all plates.

(d) Beam Data

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
1	<u>Overall beam card (Only one card for entire structure)</u>	01-12	Young's Modulus for beams (psi)	E12.4
		13-24	Shear modulus for beams (psi)	E12.4
2	<u>Beam card</u>	01-04	Beam number (must start with 1 and be in sequence)	I 4
		05-08	Node 1 ($X = 0$)	I 4
		09-12	Node 2 ($X = L$)	I 4
		13-16	Node 3—Defines positive Z orientation for straight beams and plane of curvature for curved beams. If zero, Z is parallel to structural z for straight beams. For clarification, see figure 7.	I 4
		17-20	Section property option	I 4
			0—Repeat previous beam data (section properties)	
			1—Constant cross section	
			2—Cross section varies according to formula	
			3—Cross section varies according to input table	
		21-24	Number of section properties to be defined. (Leave blank if section property option = 0 or 1.)	I 4
		25-28	Print option	I 4
			0—No print	
			1—Print coordinate-transformation, stiffness, and stress matrices for this beam element.	

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
2 (Cont.)	Beam card	29-32	Offset control	I 4
			0—Neutral axis on nodes	
			1—Neutral axis offset from nodes	
		33-36	Blank	4 X
		37-40	Curve beam option	I 4
			0—Straight beam	
			1—Curved beam	
		41-46	End fixity controls	6 I 1
			0—Fixed	
			1—Pinned	
			Order of degrees of freedom— m_X , m_Y , and m_Z at node 1; m_X , m_Y , and m_Z at node 2	
		47-48	Geometric stiffness matrix option for this beam	I 2
			0—None used	
			1—Being used; read card 2A	
		49-60	Young's modulus if different from overall value, otherwise zero (psi)	E12. 4
		61-72	Shear modulus, if different from overall value; otherwise zero (psi)	E12. 4
2A	<u>Geometric stiffness-matrix input</u> (Omit this card if card 2, columns 47 and 48, are either blank or zero.)	01-12	Axial load in the local X direction applied on the beam (lb)(tension positive)—Use this field if rotation will occur about the local Y axis.	E12. 4

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
2A (Cont.)	<u>Geometric stiffness-</u> <u>matrix input</u>	13-24	Axial loads in the local X direction applied on the beam (lb) (tension positive)—Use this field if rotation will occur about the local Z axis.	E12. 4
2B	<u>Radius of curvature</u>	01-12	Radius of curvature for neutral axis (inches) (For curved beams only. See figure 7.)	E12. 4
3	<u>Offsets</u> (in structural coordinates) (If offset control on card 2 is zero, omit this card.)	01-12 13-24 25-36 37-48 49-60 61-72	Distance of offset in x direction at node 1 (in.) Distance of offset in y direction at node 1 (in.) Distance of offset in z direction at node 1 (in.) Distance of offset in x direction at node 2 (in.) Distance of offset in y direction at node 2 (in.) Distance of offset in z direction at node 2 (in.)	E12. 4 E12. 4 E12. 4 E12. 4 E12. 4 E12. 4
4	<u>Beam properties</u> (If section property option on card 2 is zero, omit this card.)			
4A	<u>Constant cross section</u> (Section property option = 1.)	01-12	I_Y moment of inertia about local Y axis (in. ⁴)	E12. 4
		13-24	A_{XY} shear area in XY plane (sq in.) (See figure 7.)	E12. 4
		25-36	A area carrying axial load (sq in.)	E12. 4

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
4A (Cont.)	<u>Constant cross section</u>			
		37-48	I_Z moment of inertia about Z axis (in. ⁴)	E12.4
		49-60	A_{XZ} shear area in XZ plane (sq in.) (See figure 7.)	E12.4
		61-72	J torsional resistance parameter (in. ⁴)	E12.4
		Section properties may be zero or positive; cannot be negative. Zero shear area means ignore shear information.		
4A	<u>Section properties calculated according to formula (section property option = 2)</u>	01-04	Property number	I 4
		1— I_Y		
		2— A_{XY}		
		3—A		
		4— I_Z		
		5— A_{XZ}		
		6—J		
		05-08	K (See formula below.)	I 4
		09-12	L (See formula below.)	I 4
		13-16	M (See formula below.)	I 4
		17-20	N (See formula below.)	I 4
		21-24	Leave blank	4 X
		25-36	A (See formula below.)	E12.4
		37-48	B (See formula below.)	E12.4

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
4A (Cont.)	<u>Section properties calculated according to formula</u>	49-60	C (See formula below.) $f(x) = \left[1 + A \left(\frac{X}{b} \right)^K \right]^M \left[1 + B \left(\frac{X}{b} \right)^L \right]^N C$ <p>where b is the length of the beam and X is measured in inches from node 1. Note that C is the value of the section property at node 1. If section property is zero, this card may be omitted. Otherwise, properties must be in ascending order of property number. Possibly six sets of cards follow; number of sets = number of section properties to be described by input tables. For each set:</p>	E12. 4
4A	<u>Section properties vary according to input table (section property option = 3)</u>			
	<u>First card</u>	01-04	Property number (see above)	I 4
		05-08	Table length—maximum of 50	I 4
		09-12	Leave blank	4 X
		13-24	Base length—length on which X's in table are based	E12. 4
4B	<u>Second and following cards</u>	01-12, 25-36,		
		49-60	X measured from node 1 (in.)	E12. 4
		13-24,		
		37-48,		
		61-72	Corresponding value of section property	E12. 4

Repeat cards 2 through 4 for all beams.

(e) Eigenvalue Data

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
1	<u>Size card</u>	01-10	Matrix size—100 maximum (90 maximum for phase II compatibility)	I 10
		11-20	Number of modes to be found—25 maximum	I 10
2	<u>Mass card(s)</u>	01-10,	Diagonal terms of mass matrix(lb·sec ² /in.)—One	7E10.0
		11-20,	entry for each retained freedom (in the order of	
		etc.	node numbering)	

(2) Phase II Data

Phase II input is from both tape and cards. The phase I output tape must be mounted on logical unit 10 for use in phase II. Additional information, which comes from data cards, is organized into two sections: control and excitation descriptions.

The first input section (control) contains analysis and response solution options and problem-size parameters. Damping values and frequency information are also included in this section.

The second input section (excitation description) contains data necessary for the generation of force cross-PSD matrices for a panel. Care must be taken so that the order of terms in these matrices duplicates the order in the structural matrices. Compatibility can be obtained by taking the following steps:

- (a) Position the panel in the first quadrant of a rectangular coordinate system such that the first structural node with retained deflection is nearest the origin (figure 9). Indicate by vector c_t the direction of progressive wave propagation across the panel relative to this orientation.
- (b) Determine c_x and c_y , the x- and y-direction phase velocities. Signs of c_x and c_y indicate the direction of c_t (figure 10).
- (c) Determine the direction (x or y) in which the structural nodes are ordered by an increasing node number. This direction specifies the cyclic direction of node numbering (figure 11).
- (d) Assign an origin-to-line distance for each line of nodes (figure 12). Distances for two lines defining panel boundaries for each direction must be included.

Pressure PSD must be read in for each frequency at which force cross PSD is to be generated. These frequencies depend on the analysis and response options being used. For analysis option 1 and response cross-PSD solutions, pressure PSD's correspond to the frequencies specified in the first input section. When joint response moments are calculated in analysis option 3, pressure PSD's must be specified for the system's natural frequencies used in the analysis. For an option 2 joint-moment calculation, PSD's are read in two sets. The first set is for the natural frequencies and the second set is for the cross-modal terms. These cross-modal-term frequencies are in the order

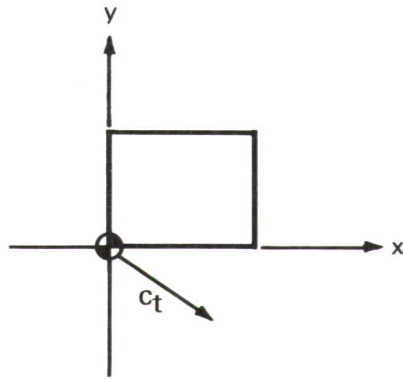


Figure 9. Panel Position

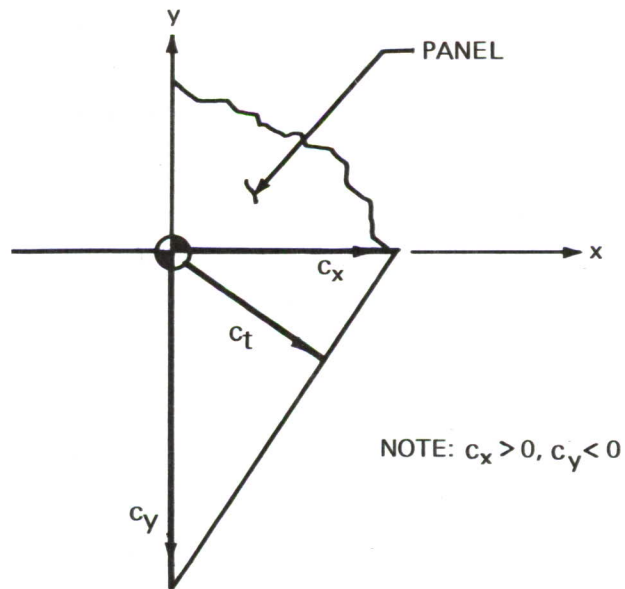


Figure 10. Phase Velocities

$$\omega_{ij} = \frac{\omega_i + \omega_j}{2}$$

$$i = 1, 2, \dots, m-1$$

$$j = i+1, \dots, i+K \leq m$$

where K is an input parameter defining the number of cross terms used in the calculation and m is the number of natural frequencies used in the analysis.

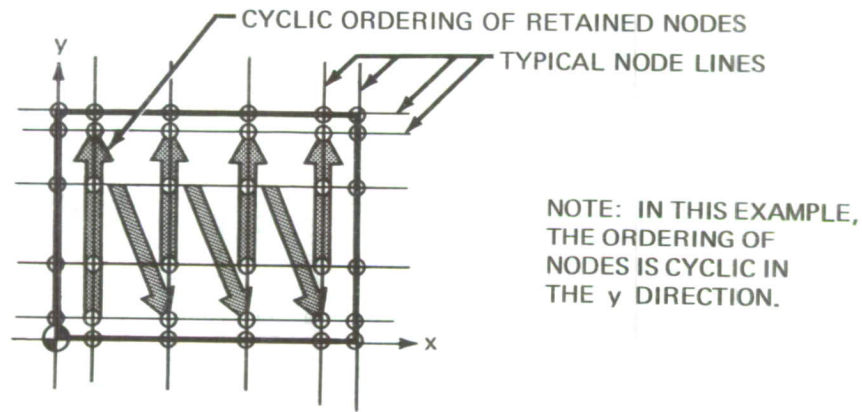


Figure 11. Cyclic Ordering of Nodes

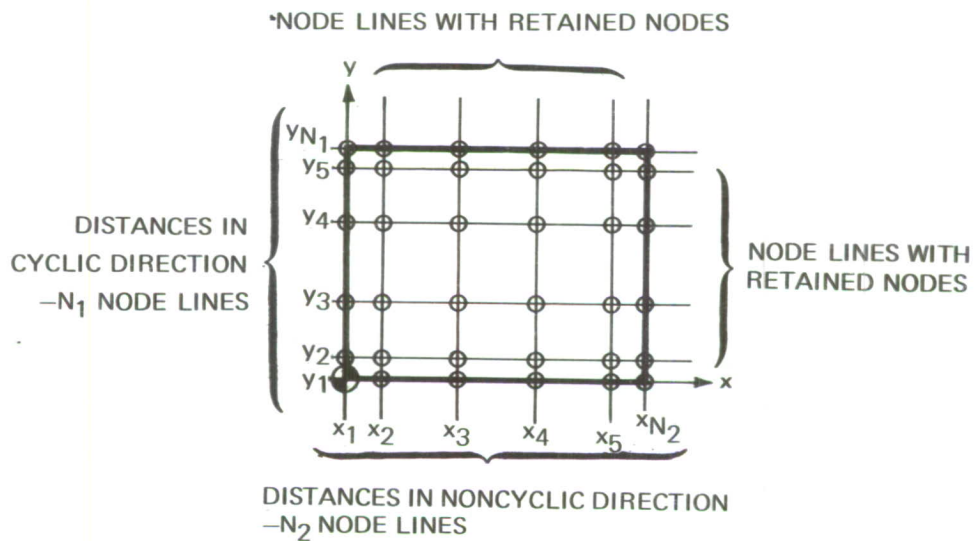


Figure 12. Node Line Distances

The following shows the input-data preparation procedure for phase II:

(a) Control

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
1	<u>Option control card</u>			
		01-10	Analysis option	I 10
		1	1—General analysis	
		2	2—Normal modes	
		3	3—Normal modes, cross terms omitted	
		11-20	Solution option	I 10
		1	1—Joint moments response	
		2	2—Cross-PSD response	
		21-30	Stress option	I 10
		0	0—No stresses	
		1	1—Compute stresses	
		31-40	Second-spectral-moment option	I 10
			(Use if columns 20 and 30 = 1.)	
		0	0—No second spectral moments	
		1	1—Compute stress second spectral moments	
		41-50	Number of plate elements for which stresses are to be computed	I 10
		51-60	Number of beam elements for which stresses are to be computed	I 10
2	<u>Parameter card</u> (Use first three fields if column 10, card 1, is 2 or 3.)	01-10	Stiffness proportional damping factor; see equation (53).	F10.0

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
2 (Cont.)	<u>Parameter card</u>			
		11-20	Mass proportional damping factor; see equation (53).	F10.0
		21-30	Structural damping coefficient; see equation (54).	F10.0
		31-40	Number of retained freedoms—90 maximum (60 maximum if option 1 analysis)	I 10
		41-50	Number of cross product terms— K for which $ j - k \leq K$ in equations (62) and (63) (Use if column 10, card 1 = 2.)	I 10
		51-60	Number of frequencies for which cross PSD is to be computed (90 maximum) (Use if column 10, card 1 = 1, or if column 20, card 1 = 2.)	I 10
3	<u>Frequency card(s)</u> (Omit this card unless column 10, card 1, is 1 or column 20, card 1, is 2)	01-10, 11-20, etc.	Frequencies for which cross PSD is to be computed (rad/sec)	7F10.0

(b) Excitation Description

Card No.	Card title	Column	Description	Format
1	<u>Title card</u>	01-80	Title	14A6
2	<u>Option control card</u>	01-10	Nodal area option	I 10
			1—Read areas	
			2—Calculate areas	
		11-20	Sonic pressure wave	I 10
			1—Normal incidence wave	
			2—Progressive wave	
		21-30	Number of frequencies for which force cross-PSD matrices are to be printed	I 10
		31-40	Direction in which node numbering is cyclic	I 10
			1— y direction	
			2— x direction	
		01-10	Decay constant; see equations (22) and (23)	F10.0
3	<u>Parameter card</u>	11-20	c _x phase velocity of pressure wave along panel in x direction (Leave blank when this phase velocity is infinite.)	F10.0
		21-30	c _y phase velocity of pressure wave along panel in y direction (Leave blank when this phase velocity is infinite.)	F10.0

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
4	<u>Grid-size card</u>	01-10	Number of coordinates (including two boundaries) in the direction of cyclic node numbering	I 10
		11-20	Number of coordinates (including two boundaries) in the direction perpendicular to the cyclic node numbering direction	I 10
5	<u>Coordinate card(s)</u>	01-10, 11-20, etc.	Set of origin-to-node line distances (in.) in cyclic direction (include two boundaries)	7F10.0
6	<u>Coordinate card(s)</u>	01-10, 11-20, etc.	Set of origin-to-node line distances (in.) perpendicular to cyclic direction (include two boundaries)	7F10.0
7	<u>Nodal area card(s)</u> (Omit this card unless column 10, card 2, is 1.)	01-10, 11-20, etc.	Area for each retained node (sq in.), in the order of node numbering	7F10.0
8	<u>PSD card(s)</u>	01-10, 11-20, etc.	Pressure-PSD [(psi) ² (sec)] for each frequency at which force cross PSD is to be computed (Two sets if joint moments solution and option 2 analysis)	7F10.0

(c) Damping Matrix (Use this section only for a general analysis option 1.)

<u>Card No.</u>	<u>Card title</u>	<u>Column</u>	<u>Description</u>	<u>Format</u>
1	<u>Damping card(s)</u>	01-10, 11-20, etc.	Matrix of viscous damping coefficients	7F10.0

4. OUTPUT DATA

a. Standard

Input data for the structural matrix generation module in phase I are printed as they are read. The generated reduced stiffness matrix is printed only if the print option is exercised.

Flexibility, dynamic, and input mass matrices are printed in the eigenvalue module. After eigenvalues and eigenvectors have been determined, matrix $\{[\phi]^T [M][\phi]\}$ is printed. Then, diagonal elements of this matrix (i.e. generalized masses) are printed separately. The final output of the first phase is the mode-shape vectors and natural frequencies.

Ordering of terms in the output matrices corresponds to structural node numbering, whereby the first element corresponds to the retained freedom with the lowest node number. If more than one freedom is retained for a node, displacements for that node are in the order $(\theta_x, \theta_y, \theta_z, \delta_x, \delta_y, \delta_z)$.

In phase II, input data specifying solution options and parameters are printed as they are read. Also, natural frequencies and generalized masses are printed as they are read from the phase I tape. Printing of generated force cross-PSD matrices is optional.

Further output is dependent upon the solution and analysis options selected. Either deflection cross-PSD matrices are printed for each frequency specified or the deflection covariance matrix is the output. For analysis options 1 and 2, real and imaginary parts are printed for these matrices, whereas only real parts are printed in option 3. Deflection response matrices are of order n by n , whereby n is the number of freedoms retained in the analysis.

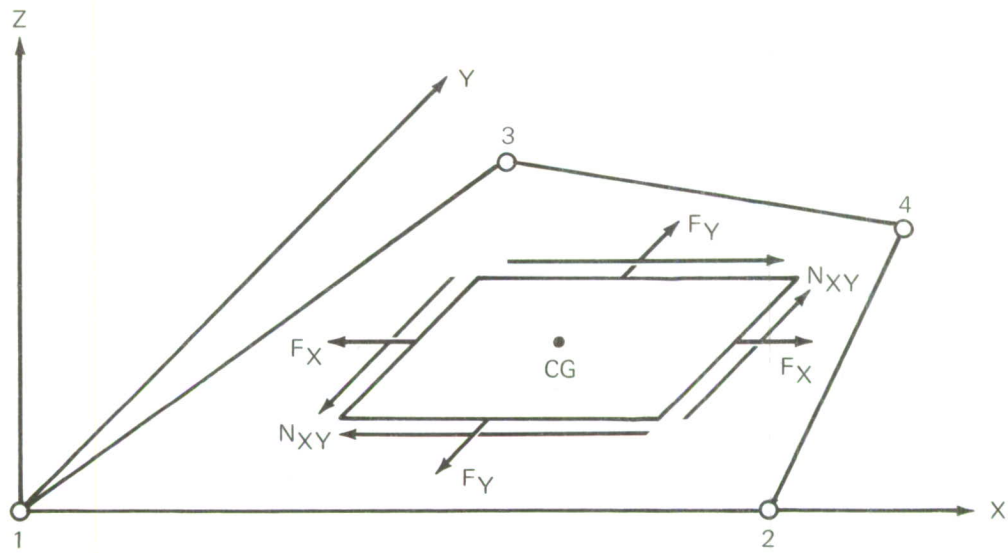
Ordering of deflections in the matrices again corresponds to the structural node numbering.

Stress cross-PSD or joint moment matrices are printed if the stress response option is exercised. Stress response matrices are printed for individual structural members. For each plate, an 8-by-8 matrix is printed with elements corresponding to stresses (F_X , F_Y , N_{XY} , Q_X , Q_Y , M_X , M_Y , M_{XY}). These stress components (figure 13) refer to forces per linear inch at a section through the center of gravity of the plate element. Two 6-by-6 matrices are printed for each beam element, one each for the stresses at each end of the beam. Beam stresses (figure 14) correspond to end forces (M_X , M_Y , M_Z , F_X , F_Y , F_Z).

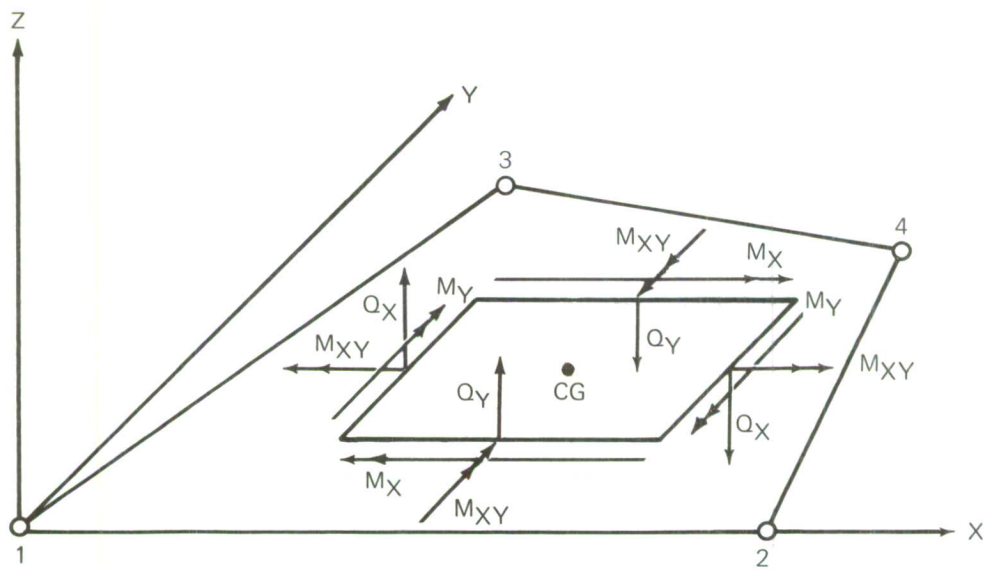
b. Diagnostics

Error messages possible in the structural generation module are as follows:

- (1) NODE XX OUT OF SEQUENCE —Indicates bad nodal data; check data deck.
- (2) INCORRECT NODE NUMBER, PLATE NO. XX—Indicates plate's node number is greater than the number of nodes in the structure; check data deck.
- (3) NEGATIVE MODULUS OF ELASTICITY, PLATE NO. XX—Indicates negative Young's modulus for plate; program converts value to positive.
- (4) NEGATIVE POISSON RATIO, PLATE NO. XX—Indicates negative Poisson ratio for plate; program converts value to positive.
- (5) INPUT NOT IN PROPER SEQUENCE, PLATE NO. XX SHOULD BE XX—Indicates bad plate data; check data deck.
- (6) NEGATIVE MOMENT OF INERTIA, PLATE NO. XX—Indicates negative moment of inertia for plate; program converts value to positive.
- (7) NEGATIVE THICKNESS, PLATE NO. XX—Indicates negative t_o , t_X , t_Y , or t_S ; program converts value to positive.
- (8) QUADRILATERAL PLATE NO. XX—Has a re-entrant corner. Local coordinates are: X coordinates (XX, XX, XX); Y coordinates (XX, XX, XX). Indicates plate has an interior angle greater than 180 degrees; if data are valid, then represent the quadrilateral as two triangles.

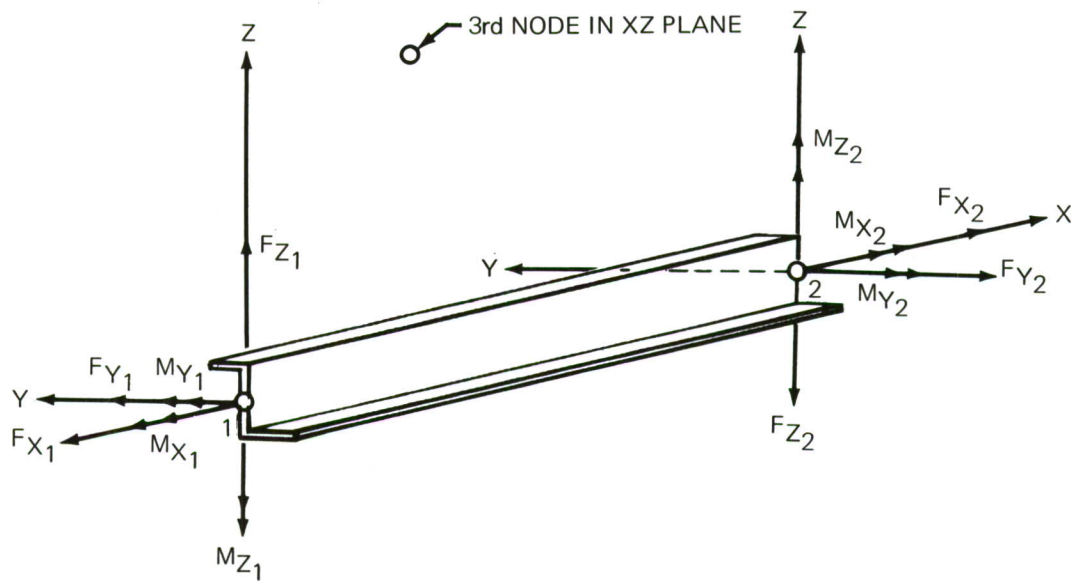


(a) Positive In-Plane Plate Stresses Per Linear Inch At CG Of Plate

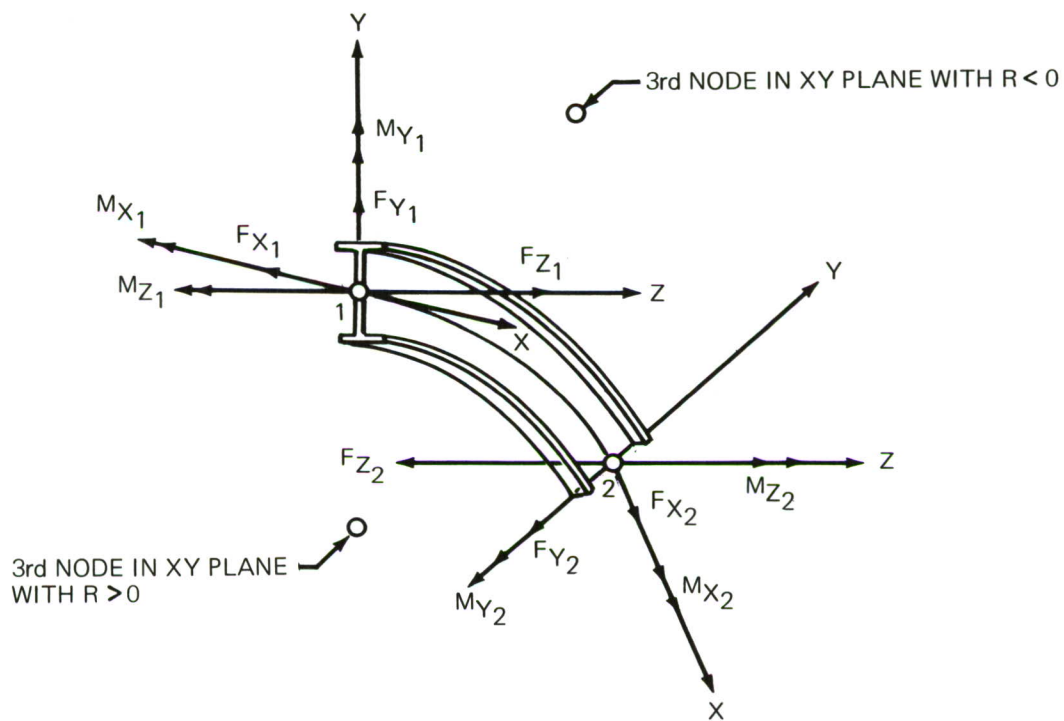


(b) Positive Out-Of-Plane Plate Stresses Per Linear Inch At CG Of Plate

Figure 13. Plate Stresses



(a) Positive Beam Stresses—Straight Beams



(b) Positive Beam Stresses—Curved Beams

Figure 14. Beam Stresses

- (9) QUADRILATERAL PLATE NO. XX/DEGREE OF NONCOPLANARITY XX PERCENT/ACCEPTABLE-ANALYSIS CONTINUED (larger than 0.1 percent) — Indicates the four plate nodes are not coplanar, but by an acceptable amount; check coordinates of the nodes.
- (10) QUADRILATERAL PLATE NO. XX/DEGREE OF NONCOPLANARITY XX PERCENT/NOT ACCEPTABLE-ANALYSIS DELETED— Indicates that the four plate nodes are not coplanar by an unsatisfactory amount; check coordinates of the nodes.
- (11) INCORRECT NODE NUMBER, BEAM NO. XX — Indicates beam's node number is greater than the number of nodes in the structure.
- (12) NEGATIVE MODULUS OF ELASTICITY, BEAM NO. XX — Indicates negative Young's Modulus for this beam; program converts value to positive.
- (13) NEGATIVE MODULUS OF RIGIDITY, BEAM NO. XX — Indicates negative shear modulus for this beam; program converts value to positive.
- (14) INPUT NOT IN PROPER SEQUENCE, BEAM NO. XX SHOULD BE XX — Indicates bad beam data; check data deck.
- (15) SECTION PROPERTY XX IS OUT OF SEQUENCE — Indicates section properties for this beam are not sequential; re-order section property data.
- (16) THE NUMBER OF PARTITIONS IN K(FF) HAS BECOME GREATER THAN 800 IN THE UNKNOWN DEFLECTION COMPUTATIONS — Indicates too many stiffness partitions have been created; reduce size of structure, if possible.
- (17) MATRIX IS SINGULAR — Indicates a diagonal K partition is singular; check data (Either the structure is unstable, or some orientation nodes are not constrained.)

The following diagnostic messages from the eigenvalue module indicate errors in reading and writing tape.

- (1) FLEXIBILITY MATRIX COULD NOT BE FOUND
CURRENT TAPE IN USE = XX, ERROR CODE = XX

- (2) SPACING ERROR OCCURRED WHILE TRYING TO WRITE ON BINARY TAPE.
 NFILE = XX, NMAT = XX
 CURRENT TAPE IN USE = XX. ERROR CODE = XX
- (3) ERROR WHILE { READING MASS MATRIX } ON TAPE
 { WRITING DYNAMIC MATRIX }
- CURRENT TAPE IN USE = XX, ERROR CODE = XX.

In the preceding diagnostics, the logical tape number is given, NFILE is the number of file marks, and NMAT is the number of matrices. Error codes are as follows:

- (1) File spacing error
- (2) Negative matrix spacing
- (3) Matrix spacing error
- (4) Check sum error
- (5) Tape name wrong

Other possible error messages are:

- (1) EIGENVALUE = XX FOR MODE XX, HAS A NEGATIVE VALUE, WHICH DOES NOT DESCRIBE A PROPER PHYSICAL SYSTEM.
- (2) MODE HAD ONLY XX PERCENT ACCURACY, WHICH IS UNDER THE GIVEN LIMIT OF XX. THE TWO VECTORS WHICH SHOULD BE IDENTICAL ARE GIVEN BELOW.

XXXXXX	1	XXXXXX
.	2	.
.	3	.
.	.	.
.	N	.

IV

SAMPLE PROBLEMS

This section describes the application of RANVIB to random structural response analysis. Example problems are given for two structures: a simply supported beam and a sandwich plate. System input and output data described in this section are given in appendix III.

1. SIMPLY SUPPORTED BEAM

The first example illustrates solution options of the program on a very simple structure. A simply supported beam is shown schematically in figure 15 together with its properties and structural idealization. The beam is loaded by a random pressure wave convected along the beam in the x direction; its characteristics are shown in the figure. Deflection and stress responses of the beam are sought for this loading.

Input data for phase I are given in appendix III, page 85. Columns 79 and 80 are used in these examples to identify sections and cards defined in the input-preparation description. The beam is idealized into six equal elements, and beam bending is the only deformation considered. Pressure is applied normal to the beam's surface, and rotary inertia effects are assumed to be negligible. Therefore, only transverse deflections are retained explicitly in the analysis. Four mode shapes and frequencies are to be determined in this example.

Printed results of the phase I analysis are presented in appendix III, pages 86 through 96.

Several phase II solution options are presented for the beam problem to illustrate their use. The analyses performed are: (1) option 3 joint moments, (2) option 2 joint moments, (3) option 3 cross PSD, (4) option 2 cross PSD, and (5) option 1 cross PSD.

Input data for the option 3 joint-moment solution are given in appendix III, page 97. Deflection and stress covariance and second spectral moment matrices are to be computed. Structural damping is assumed with a damping coefficient of 0.010. Thus, the critical damping ratio is 0.005 for all frequencies considered. The force cross-PSD matrix is printed only for the first natural frequency. Coordinates across the width of the beam were selected

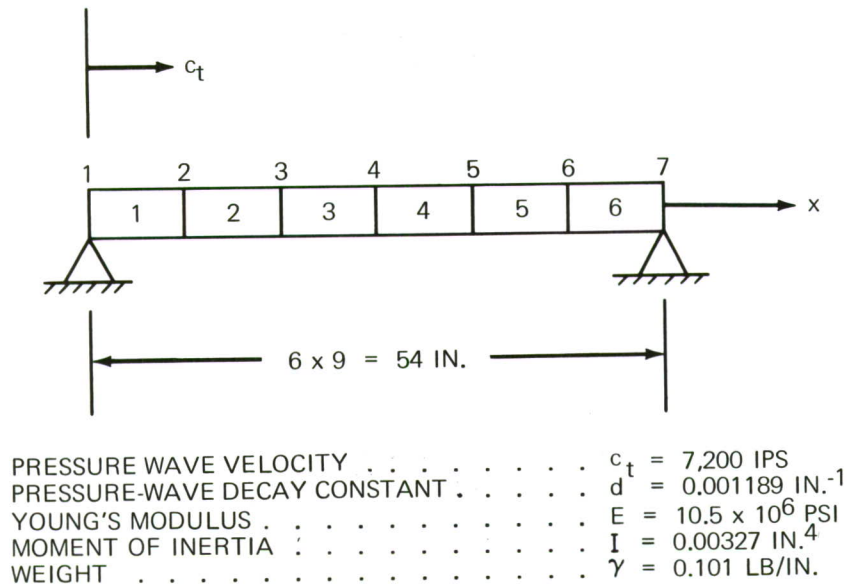


Figure 15. Simply Supported Beam

such that the nodal areas calculated are in unity. Pressure PSD is assumed to be in unity at each of the four natural frequencies used in the analysis.

Analysis results are presented in appendix III, page 98. Diagonal terms of the deflection covariance matrix are mean-square values of the modal deflections. In the stress covariance matrices, values are obtained only for M_Y and F_Z components, because twisting, axial, and out-of-plane deformations are not considered. Elements of the second spectral moment matrices can be used in equation (47) to predict the number of zero crossings of response time history. As an example, this number for the shearing force F_Z at the end of the beam is

$$\begin{aligned}
 N_o &= \frac{1}{\pi} \left[\frac{2.316 \times 10^8}{3.498 \times 10^4} \right]^{1/2} \\
 &= 25.9/\text{sec}
 \end{aligned}$$

Thus, the "apparent" frequency is

$$\frac{1}{2} N_o = 13 \text{ Hz}$$

For the moment at the beam center, this frequency is

$$\frac{1}{2} N_o = \frac{1}{2\pi} \left[\frac{1.801 \times 10^{10}}{1.046 \times 10^7} \right]^{1/2} = 6.6 \text{ Hz}$$

Input data for the option 2 joint-moments analysis are shown in appendix III, page 111. One cross product for each mode is to be included in the calculations. Pressure PSD for each of the cross-term frequencies $\left(\frac{\omega_1 + \omega_2}{2}, \frac{\omega_2 + \omega_3}{2}, \text{ and } \frac{\omega_3 + \omega_4}{2} \right)$ is also assumed to be unity.

Printed analysis results are presented in appendix III, page 112. In addition to force cross PSD being printed for the first modal frequency, force co-PSD and quad-PSD matrices are printed for the first cross modal term. The real part of the response is not significantly different from that obtained in option 3. For this problem in which damping is very low and modal frequencies are well separated, the cross terms do not contribute significantly to the result. Imaginary parts of the joint-moment matrices indicate phase differences between responses.

An illustration of input data for option 3 cross-PSD calculation is given in appendix III, page 137. Both deflection and stress cross PSD's are to be computed for one frequency: the first natural frequency of the beam. Output results are presented in appendix III, page 138.

Input data for an option 2 analysis for the preceding problem is given in appendix III, page 148. Printed results for this analysis are presented in appendix III, page 149.

The final example given for the beam problem is an option 1 cross-PSD analysis for the same frequency used in the previous solutions. Input data are shown in appendix III, page 166. For this illustrative problem, the matrix of viscous damping coefficients has been selected to be proportional to the stiffness matrix. Normally, option 1 would not be used in this case, but these coefficients were chosen so that the damping ratio at the first natural frequency is equal to that used in previous solutions. Thus, the results of this analysis can be compared to the other options.

Printed results of the analysis are presented in appendix III, page 167. Computed response is essentially the same as in other options. If joint statistical moments are computed in option 1, the response spectrum must be determined at a sufficient number of frequencies since a numerical integration is performed to calculate joint moments.

2. SANDWICH PLATE

The second example problem is for a sandwich plate simply supported on two opposite sides and supported on beams on the other two sides. This example is offered as an illustration of phase I input data for more-complex structures.

The structure is shown schematically in figure 16. Plate and beam properties are also shown in the figure. The neutral axis of each beam is 1 inch below the plane of the plate.

Structural idealization used in the problem is shown in figure 17. Input data for phase I are presented in appendix III, page 183. All in-plane deformations are assumed to be negligible and, thus, are not considered. Shear deformation of the sandwich core material is also neglected. Only bending and torsional distortions are considered for the beams. Transverse deflections at the nodes not rigidly supported are retained in the analysis. Four modal frequencies are to be determined.

The printed phase I output for the problem is presented in appendix III, page 186.

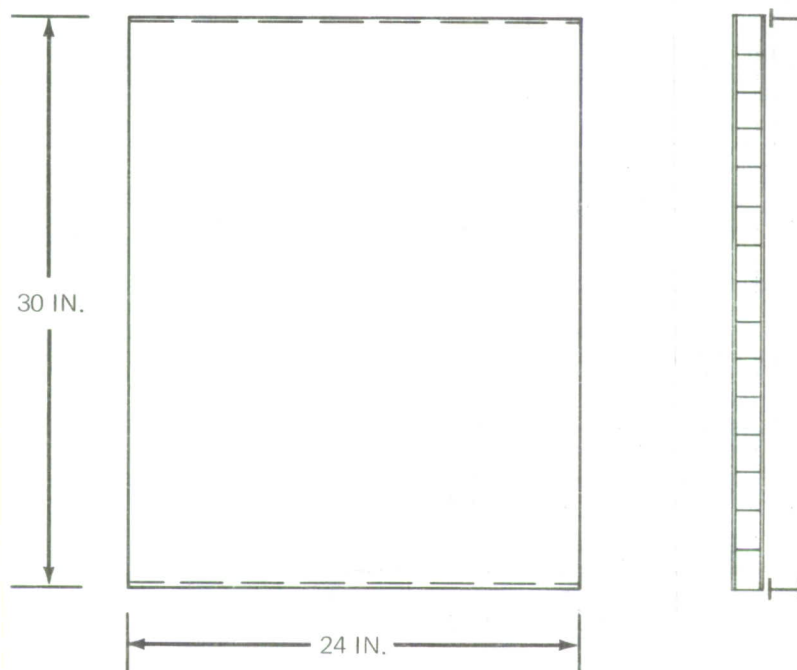


PLATE PROPERTIES:

YOUNG'S MODULUS	$E = 10.5 \times 10^6 \text{ PSI}$
POISSON'S RATIO	$\nu = 0.33$
PLATE (SKIN) THICKNESS	$t_o/2 = 0.02 \text{ IN.}$
SANDWICH DEPTH	$d = 0.5 \text{ IN.}$
WEIGHT	$\gamma = 0.007 \text{ PSI}$

BEAM PROPERTIES:

YOUNG'S MODULUS	$E = 10.5 \times 10^6 \text{ PSI}$
SHEAR MODULUS	$G = 3.9 \times 10^6 \text{ PSI}$
MOMENT OF INERTIA	$I_y = 0.1 \text{ IN.}^4$
TORSION CONSTANT	$J = 8 \times 10^{-5} \text{ IN.}^4$
WEIGHT	$\gamma = 0.016 \text{ LB/IN.}$

Figure 16. Sandwich Plate

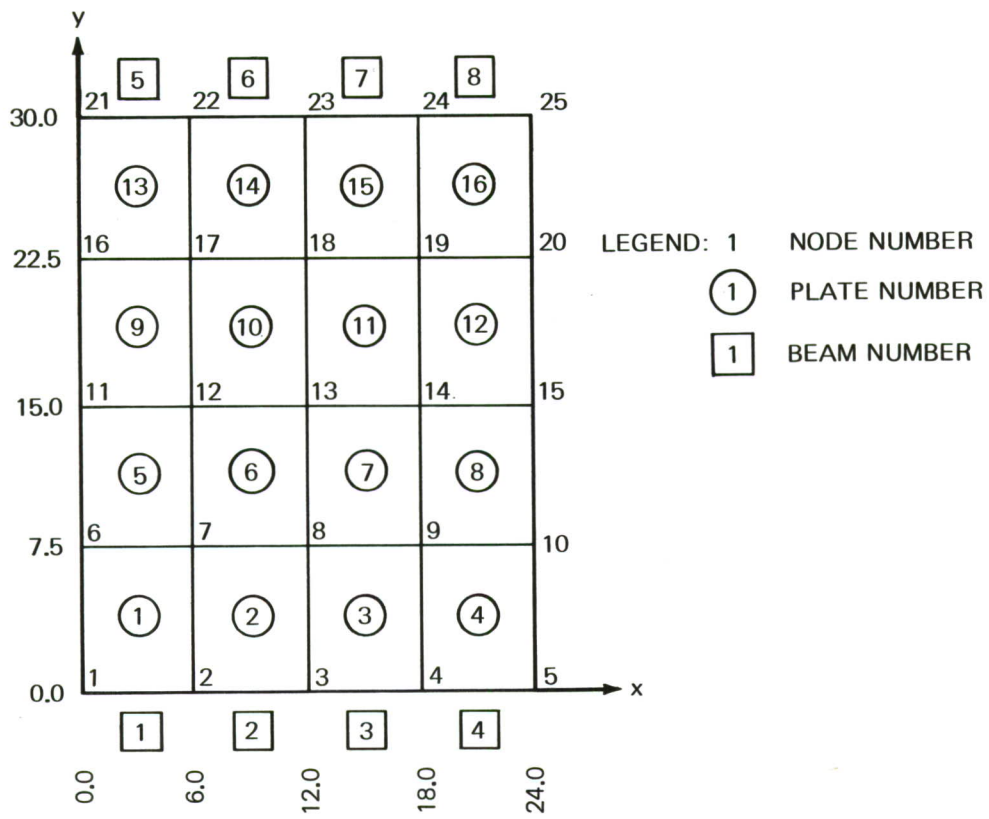


Figure 17. Idealization of Sandwich Plate

APPENDIX I

STRUCTURAL ELEMENTS

1. A BRIEF DISCUSSION OF ELEMENT STIFFNESS MATRICES

As implied in the input description, the structural matrix module has all of the standard elements of any displacement program, i.e., the rod, the beam, and a plate capable of carrying in-plane and/or out-of-plane loading.

The element flexibility matrix for the curved beam is obtained using the principle of complementary energy. This matrix is then inverted to give the element stiffness matrix. The element stiffness matrix for the straight beam is obtained from that for the curved by considering the limit as the radius approaches infinity. The rod stiffness is simply that for the straight beam, assuming no bending resistance.

Matrices for quadrilateral plate elements are generated by first subdividing the elements into four triangles.

The triangular plate element is basically that found in reference 9. Out-of-plane bending stiffness is obtained by "sandwiching" two identical in-plane plates together. The plate does have the unique characteristic of having an in-plane moment stiffness. This is used to prevent singularities when a user does not wish to support this freedom.

For a detailed description of the elements in this program, see reference 4.

2. GEOMETRIC STIFFNESS MATRICES

The program contains geometric stiffness matrices for the following:

- a. Beam — axial load in local X direction for rotation about the local Y axis, or rotation about the local Z axis
- b. Triangular plate — in-plane stress in the local X direction
- c. Triangular plate — in-plane stress in the local Y direction
- d. Triangular plate — in-plane pure shear stress

For a description of the use and derivation of these elemental matrices, see reference 6.

Figure 18 shows the matrix elements for a beam of length L with tensile force F_X and rotation about the local Y axis. This matrix can be modified for rotation about the local Z axis by shifting rows and columns 3, 5, 9, and 11 to rows and columns, 2, 6, 8, and 12, respectively.

Geometric stiffness matrices for a triangular plate, such as those shown in figure 7(b), with in-plane tensile stresses σ_X and σ_Y and shear stress τ_{XY} are shown in figure 19. The thickness of the plate is t and $X_{32} = X_3 - X_2$.

3. INTERPRETATION OF PLATE INPUT DATA

For a uniform and solid plate, the actual thickness t_o is read from the plate card. Other plate properties are computed by the program as follows:

$$\begin{aligned} t_X &= 0 & I_X &= 0 \\ I_Y &= 0 & t_s &= t_o \\ t_Y &= 0 & I_s &= t_o^3/12 \end{aligned}$$

The values of t_o and t_s must be positive. For negative values, the program takes the absolute value and prints a warning.

For stiffened or sandwich plates, values for the properties in addition to t_o are read from card 3 of the plate data. Stiffener properties t_X , I_Y , t_Y , and I_X can be zero. If the effective shear thickness t_s is read in as zero, the program sets t_s equal to t_o . If the plate bending moment of inertia I_s is read in as zero, the program sets $I_s = t_o^3/12$. The quantities t_o , t_X , I_Y , I_X , t_s , and I_s must not be negative. For negative values, the program takes the absolute value and prints a warning.

The stiffened-plate option is one alternative when structure can be idealized into an assembly of equivalent orthotropic plate elements. The model for this plate element uniformly distributes equivalent stiffener properties into the plate. An alternate procedure with better results when stiffeners are widely separated is to use beam elements to represent the stiffeners.

$$\begin{bmatrix} F_{X1} \\ F_{Y1} \\ F_{Z1} \\ \hline M_{X1} \\ M_{Y1} \\ M_{Z1} \\ \hline F_{X2} \\ F_{Y2} \\ F_{Z2} \\ \hline M_{X2} \\ M_{Y2} \\ M_{Z2} \end{bmatrix} = F_X \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6/5L & -1/10 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1/10 & 0 & 2L/15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -6/5L & 1/10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/10 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{X1} \\ \delta_{Y1} \\ \delta_{Z1} \\ \hline \theta_{X1} \\ \theta_{Y1} \\ \theta_{Z1} \\ \hline \delta_{X2} \\ \delta_{Y2} \\ \delta_{Z2} \\ \hline \theta_{X2} \\ \theta_{Y2} \\ \theta_{Z2} \end{bmatrix}$$

Figure 18. Geometric Stiffness Matrix for a Beam
(Rotation about Local Y Axis)

$$\begin{Bmatrix} F_{X1} \\ F_{Y1} \\ F_{Z1} \\ F_{X2} \\ F_{Y2} \\ F_{Z2} \\ F_{X3} \\ F_{Y3} \\ F_{Z3} \end{Bmatrix} = \frac{t \sigma_X}{2X_2Y_3} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3^2 & 0 & 0 & Y_3^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -Y_3^2 & 0 & 0 & Y_3^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \delta_{X1} \\ \delta_{Y1} \\ \delta_{Z1} \\ \delta_{X2} \\ \delta_{Y2} \\ \delta_{Z2} \\ \delta_{X3} \\ \delta_{Y3} \\ \delta_{Z3} \end{Bmatrix}$$

$$\begin{Bmatrix} F_{X1} \\ F_{Y1} \\ F_{Z1} \\ F_{X2} \\ F_{Y2} \\ F_{Z2} \\ F_{X3} \\ F_{Y3} \\ F_{Z3} \end{Bmatrix} = \frac{t \sigma_Y}{2X_2Y_3} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & X_{32}^2 & 0 & 0 & -X_{32}X_3 & 0 & 0 & X_{32}X_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -X_{32}X_3 & 0 & 0 & X_3^2 & 0 & 0 & -X_3X_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & X_{32}X_2 & 0 & 0 & -X_3X_2 & 0 & 0 & X_2^2 \end{bmatrix} \begin{Bmatrix} \delta_{X1} \\ \delta_{Y1} \\ \delta_{Z1} \\ \delta_{X2} \\ \delta_{Y2} \\ \delta_{Z2} \\ \delta_{X3} \\ \delta_{Y3} \\ \delta_{Z3} \end{Bmatrix}$$

$$\begin{Bmatrix} F_{X1} \\ F_{Y1} \\ F_{Z1} \\ F_{X2} \\ F_{Y2} \\ F_{Z2} \\ F_{X3} \\ F_{Y3} \\ F_{Z3} \end{Bmatrix} = \frac{t \tau_{XY}}{2X_2Y_3} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2X_{32}Y_3 & 0 & 0 & (X_3Y_3 + X_{32}Y_3) & 0 & 0 & -X_2Y_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (X_3Y_3 + X_{32}Y_3) & 0 & 0 & -2X_3Y_3 & 0 & 0 & X_2Y_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -X_2Y_3 & 0 & 0 & X_2Y_3 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \delta_{X1} \\ \delta_{Y1} \\ \delta_{Z1} \\ \delta_{X2} \\ \delta_{Y2} \\ \delta_{Z2} \\ \delta_{X3} \\ \delta_{Y3} \\ \delta_{Z3} \end{Bmatrix}$$

Figure 19. Geometric Stiffness Matrices for a Triangular Plate

APPENDIX II

ADMITTANCE INTEGRALS

To obtain joint response moments in the normal mode analysis (option 2), it is necessary to evaluate certain integrals having system admittance terms. These integrals have been evaluated in reference 8, and results are presented in this appendix.

The admittance integrals appearing in the equation for deflection covariance, equation (63), page 27, are of the form:

$$\int_0^{\infty} (D_i D_j + E_i E_j) d\omega$$

$$\int_0^{\infty} D_i E_j d\omega$$

where D_j and E_j are defined by equations (60) and (61), page 26. For convenience in expressing the integrals, the modal damping is specified by a parameter μ_j given as

$$\mu_j = 2\zeta_j \omega_j$$

Damping is assumed to be small ($\zeta_j < 1$). Results of the integrations are

$$\int_0^{\infty} (D_i D_j + E_i E_j) d\omega =$$

$$\frac{\pi(\mu_i + \mu_j)}{M_i M_j [\omega_i^4 + (\mu_j^2 + \mu_i \mu_j) \omega_i^2 + \omega_j^4 + (\mu_i^2 + \mu_i \mu_j) \omega_j^2 - 2\omega_i^2 \omega_j^2]}$$

$$\int_0^\infty D_i E_j d\omega =$$

$$\frac{1}{2M_i M_j} \left\{ \frac{B_{ij}}{2} \log_e \frac{\omega_j^4}{\omega_i^4} + \left[A_{ij} - \frac{B_{ij}}{2} (\mu_i^2 - 2\omega_i^2) \right] \left(\frac{\pi}{\mu_i \sqrt{4\omega_i^2 - \mu_i^2}} \right) \right.$$

$$\left. + \left[C_{ij} + \frac{B_{ij}}{2} (\mu_j^2 - 2\omega_j^2) \right] \left(\frac{\pi}{\mu_j \sqrt{4\omega_j^2 - \mu_j^2}} \right) \right\}$$

where the constants are given by

$$B_{ij} = \frac{(\omega_i^4 - \omega_j^4) [-\mu_j \omega_i^2 (\mu_i^2 - 2\omega_i^2) - \mu_j \omega_i^4] + \mu_j \omega_i^2 [\omega_i^4 (\mu_j^2 - 2\omega_j^2) - \omega_j^4 (\mu_i^2 - 2\omega_i^2)]}{\omega_i^4 [\mu_i^2 - \mu_j^2 - 2(\omega_i^2 - \omega_j^2)] [\omega_i^4 (\mu_j^2 - 2\omega_j^2) - \omega_j^4 (\mu_i^2 - 2\omega_i^2)] - \omega_i^4 (\omega_i^4 - \omega_j^4)^2}$$

$$A_{ij} = \frac{B_{ij} \omega_i^4 [\mu_i^2 - \mu_j^2 - 2(\omega_i^2 - \omega_j^2)] - \mu_j \omega_i^2}{\omega_i^4 - \omega_j^4}$$

$$C_{ij} = \frac{\mu_j \omega_i^2 - \omega_j^4 A_{ij}}{\omega_i^4}$$

The expression for the second integral is valid only for $i = j$. This integral is not required for $i \neq j$, because it is multiplied by a null matrix in equation (63), page 27.

Calculation of second spectral moments requires the evaluation of integrals formed from the previous ones by multiplying the integrands by ω^2 . These integrals are given by

$$\int_0^\infty \omega^2 (D_i D_j + E_i E_j) d\omega =$$

$$\frac{\pi(\mu_i^2 \omega_j^2 + \mu_j^2 \omega_i^2)}{[M_i M_j] [\omega_i^4 + (\mu_j^2 + \mu_i \mu_j) \omega_i^2 + \omega_j^4 + (\mu_i^2 + \mu_i \mu_j) \omega_j^2 - 2\omega_i^2 \omega_j^2]}$$

$$\int_0^{\infty} \omega^2 D_{ij} E_j d\omega =$$

$$\frac{1}{2M_i M_j} \left\{ \frac{F_{ij}}{2} \log_e \frac{\omega_j^4}{\omega_i^4} + [E_{ij} - \frac{F_{ij}}{2} (\mu_i^2 - 2\omega_i^2)] \frac{\pi}{\mu_i \sqrt{4\omega_i^2 - \mu_i^2}} \right. \\ \left. + [G_{ij} + \frac{F_{ij}}{2} (\mu_j^2 - 2\omega_j^2)] \frac{\pi}{\mu_j \sqrt{4\omega_j^2 - \mu_j^2}} \right\}$$

where

$$F_{ij} = \frac{\mu_j \omega_i^6 (\omega_i^4 - \omega_j^4) + \mu_j \omega_j^4 [\omega_i^4 (\mu_j^2 - 2\omega_j^2) - \omega_i^4 (\mu_i^2 - 2\omega_i^2)]}{\omega_i^4 [\mu_i^2 - \mu_j^2 - 2(\omega_i^2 - \omega_j^2)] [\omega_i^4 (\mu_j^2 - 2\omega_j^2) - \omega_j^4 (\mu_i^2 - 2\omega_i^2)] - \omega_i^4 (\omega_i^4 - \omega_j^4)^2}$$

$$E_{ij} = \frac{F_{ij} \omega_i^4 [\mu_i^2 - \mu_j^2 - 2(\omega_i^2 - \omega_j^2)] - \mu_j \omega_i^4}{\omega_i^4 - \omega_j^4}$$

$$G_{ij} = - E_{ij} \frac{\omega_j^4}{\omega_i^4}$$

Again, it's not necessary to evaluate the second of these integrals for $i = j$.

APPENDIX III

SYSTEM INPUT AND OUTPUT DATA

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PHASE 1 SYSTEM CONTROL CARDS

```

$EXECUTE      IBJOB
$IBJOB
$POOL         BUFCT=07,'UNIT02','UNIT03','UNIT04',
$ETC          'UNIT11','UNIT12','UNIT13','UNIT14','UNIT15','UNIT16'
$ETC          'UNIT17','UNIT10','UNIT03'
$GROUP        OPNCT=07,BUFCT=07,'UNIT02','UNIT03','UNIT04',
$ETC          'UNIT11','UNIT12','UNIT13','UNIT14','UNIT15','UNIT16'
$ETC          'UNIT17','UNIT10','UNIT03'
$IEDIT        SYSLB4,SRCH
$IBLDR BLK
$IBLDR PHASE1
$IBLDR WRTETP
$IBLDR READTP
$IBLDR FVIO
$IBLDR FILE
$IBLDR FBSF
$IBLDR FFSF
$IBLDR FFSR
$IBLDR FKRD
$IBLDR FRUN
$ORIGIN       PHASE1,SYSC2
$IBLDR MAST*
$IBLDR PAGH*
$IBLDR UNPAC*
$IBLDR PRINT*
$ORIGIN       ALPHA1,SYSC2
$IBLDR SUBM1*
$ORIGIN       BETA1,SYSC2
$IBLDR GENRA*
$IBLDR REDUC*
$ORIGIN       GAMMA1,SYSC2
$IBLDR INFO*
$ORIGIN       GAMMA1,SYSC2
$IBLDR PLATE*
$IBLDR MUL1*
$IBLDR MUL2*
$IBLDR PSTIF*
$IBLDR QUAD*
$IBLDR LAMK*
$IBLDR KLAMT*
$IBLDR TRI*
$IBLDR INP*
$IBLDR INPM*
$IBLDR INPST*
$IBLDR OUTP*
$IBLDR OUTPM*
$IBLDR OUTPS*
$IBLDR COMBI*
$IBLDR STORE*
$IBLDR MOVE*
$IBLDR PMTR*

```

\$IBLDR SMTR*	
\$IBLDR LOCAL*	
\$IBLDR COPLA*	
\$ORIGIN	GAMMA1,SYSCK2
\$INCLUDE	FSCN,FXP2
\$IBLDR BEAM*	
\$IBLDR TINVR*	
\$IBLDR SMULT*	
\$IBLDR MULT*	
\$IBLDR SBMTR*	
\$IBLDR SSTIF*	
\$IBLDR MAD*	
\$IBLDR SBGS*	
\$IBLDR OFST*	
\$IBLDR CSTIF*	
\$IBLDR CBMTR*	
\$ORIGIN	BETA1,SYSCK2
\$IBLDR MERGE*	
\$IBLDR SOR*	
\$ORIGIN	DELTA1,SYSCK2
\$IBLDR MERGB*	
\$ORIGIN	DELTA1,SYSCK2
\$IBLDR STRES*	
\$IBLDR MSTRE*	
\$ORIGIN	ALPHA1,SYSCK2
\$IBLDR SORCON	
\$IBLDR TEST*	
\$ORIGIN	SORT1,SYSCK2
\$IBLDR FKSRT	
\$ORIGIN	SORT1,SYSCK2
\$IBLDR KFSRT	
\$ORIGIN	SORT1,SYSCK2
\$IBLDR CNCT*	
\$ORIGIN	EXP1,SYSCK2
\$IBLDR EXPND*	
\$ORIGIN	EXP1,SYSCK2
\$IBLDR EXTRN*	
\$ORIGIN	SORT1,SYSCK2
\$IBLDR DELET*	
\$IBLDR SSORT*	
\$ORIGIN	ALPHA1,SYSCK2
\$IBLDR HELP	
\$IBLDR YTLOSB	
\$IBLDR INVERT	
\$IBLDR DATASB	
\$ORIGIN	PHASE1,SYSCK2
\$IBLDR FREMO*	
\$IBLDR WDVALV	
\$IBLDR INRPRD	
\$ORIGIN	ALPHA,SYSCK2
\$IBLDR WDHESS	
\$ORIGIN	ALPHA,SYSCK2


```
$IBLDR WDQRIT
$ORIGIN      ALPHA,SYSCK2
$IBLDR WDSORT
$ORIGIN      ALPHA,SYSCK2
$IBLDR WDVECT
$ORIGIN      ALPHA,SYSCK2
$IBLDR WDTRN1
$ORIGIN      ALPHA,SYSCK2
$IBLDR WDTRN2
$ORIGIN      PHASE1,SYSCK2
$IBLDR AMERG*
$ORIGIN      PHASE1,SYSCK2
$IBLDR SMERG*
```

PHASE II SYSTEM CONTROL CARDS

\$EXECUTE	IBJOB
\$IBJOB	
\$POOL	BUFCT=09,'UNIT01','UNIT02','UNIT03','UNIT04',
\$ETC	'UNIT08','UNIT10','UNIT11','UNIT12','UNIT13','UNIT14'
\$ETC	'UNIT15','UNIT16','UNIT17'
\$GROUP	OPNCT=09,'UNIT01','UNIT02','UNIT03','UNIT04',
\$ETC	'UNIT08','UNIT10','UNIT11','UNIT12','UNIT13','UNIT14'
\$ETC	'UNIT15','UNIT16','UNIT17'
\$IEDIT	SYSLB4,SRCH
\$IBLDR PHASE2	
\$IBLDR SCALE*	
\$IBLDR FVIO	
\$IBLDR FILE	
\$IBLDR READTP	
\$IBLDR WRTETP	
\$IBLDR FBSF	
\$IBLDR FFSF	
\$IBLDR FFSR	
\$IBLDR FKRD	
\$IBLDR FRUN	
\$ORIGIN	ALPHA,SYSCK2
\$IBLDR HELP	
\$IBLDR YTLOSB	
\$IBLDR INVERT	
\$IBLDR DATASB	
\$ORIGIN	ALPHA,SYSCK2
\$IBLDR TAPOS*	
\$ORIGIN	ALPHA,SYSCK2
\$IBLDR RANLO*	
\$ORIGIN	BETA,SYSCK2
\$IBLDR ARIA**	
\$ORIGIN	BETA,SYSCK2
\$IBLDR CONST*	
\$ORIGIN	BETA,SYSCK2
\$IBLDR NOISO*	
\$IBLDR OUTPUT	
\$ORIGIN	ALPHA,SYSCK2
\$IBLDR PEDAN*	
\$ORIGIN	ALPHA,SYSCK2
\$IBLDR PRNTA*	
\$ORIGIN	ALPHA,SYSCK2
\$IBLDR PRNTB*	
\$ORIGIN	ALPHA,SYSCK2
\$IBLDR CONS*	
\$ORIGIN	ALPHA,SYSCK2
\$IBLDR PRNTC*	

\$ORIGIN	ALPHA,SYSC2
\$IBLDR PRNTD*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR DSEC1*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR ADMN3*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR ADDMA*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR ADMN2*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR CQJD*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR SUMT*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR SUM2*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR SECM3*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR SECM2*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR ADMT3*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR ADMT2*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR CQCPS*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR SUM3*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR PRNTE*	
\$ORIGIN	ALPHA,SYSC2
\$IBLDR DSECM*	

PHASE I OUTPUT DATA -- BEAM

SIMPLY SUPPORTED BEAM

CALCULATE DEFLECTIONS AND STRESSES

STRUCTURE SIZE	BEAMS	PLATES	NODES	PARTITIONS
	6	0	7	-0

LAST NODES IN PARTITIONS
1, 7

SIMPLY SUPPORTED BEAM

NODE	NODAL DATA			Z		RETAINED FREEDOMS
	BC	X	Y			
1	101111	0.0000E-39	-0.0000E-39	-0.0000E-39	000000	
2	101110	9.0000E 00	-0.0000E-39	-0.0000E-39	000001	
3	101110	1.8000E 01	-0.0000E-39	-0.0000E-39	000001	
4	101110	2.7000E 01	-0.0000E-39	-0.0000E-39	000001	
5	101110	3.6000E 01	-0.0000E-39	-0.0000E-39	000001	
6	101110	4.5000E 01	-0.0000E-39	-0.0000E-39	000001	
7	101111	5.4000E 01	-0.0000E-39	-0.0000E-39	000000	

SIMPLY SUPPORTED BEAM

BEAM NODE		NODE		NODE		NOCE		BEAM DATA							SM MOD	
1	2	3	4	5	6	7	8	FIXITY	I (V)	A (XY)	A	I (Z)	A (XZ)	J	YG MOD	
1	1	2	-0	000000	3.270E-03	-0.000E-39	-0.000E-39	000000	3.270E-03	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	1.050E 07	-0.000E-39
2	2	3	-0	000000	3.270E-03	-0.000E-39	-0.000E-39	000000	3.270E-03	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	1.050E 07	-0.000E-39
3	3	4	-0	000000	3.270E-03	-0.000E-39	-0.000E-39	000000	3.270E-03	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	1.050E 07	-0.000E-39
4	4	5	-0	000000	3.270E-03	-0.000E-39	-0.000E-39	000000	3.270E-03	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	1.050E 07	-0.000E-39
5	5	6	-0	000000	3.270E-03	-0.000E-39	-0.000E-39	000000	3.270E-03	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	1.050E 07	-0.000E-39
6	6	7	-0	000000	3.270E-03	-0.000E-39	-0.000E-39	000000	3.270E-03	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	1.050E 07	-0.000E-39

STIFFNESS MATRIX

1	4.652160E 02	-4.478318E 02	1.956625E 02	-5.218111E 01	1.304626E 01
2	-4.478318E 02	6.608786E 02	-5.000129E 02	2.087088E 02	-5.218111E 01
3	1.956625E 02	-5.000129E 02	6.739248E 02	-5.000129E 02	1.956625E 02
4	-5.218111E 01	2.087088E 02	-5.000129E 02	6.608786E 02	-4.478318E 02
5	1.304626E 01	-5.218111E 01	1.956625E 02	-4.478318E 02	4.652160E 02

BELOW IS THE COMPLETE FLEXIBILITY MATRIX UNSCALED. THE SCALE FACTOR = 1.000000E 00

1	2.948867E-02	4.482287E-02	4.600245E-02	3.656605E-02	2.005235E-02
2	4.482287E-02	7.549111E-02	8.138892E-02	6.605480E-02	3.656605E-02
3	4.600244E-02	8.138892E-02	9.554346E-02	8.138892E-02	4.600244E-02
4	3.656605E-02	6.605479E-02	8.138892E-02	7.549111E-02	4.482287E-02
5	2.005235E-02	3.656605E-02	4.600244E-02	4.482287E-02	2.948866E-02

BELOW IS THE DYNAMIC MATRIX.

1	6.935734E-05	1.054234E-04	1.081978E-04	8.600336E-05	4.716313E-05
2	1.054234E-04	1.775551E-04	1.914267E-04	1.553609E-04	8.600335E-05
3	1.081977E-04	1.914267E-04	2.247182E-04	1.914267E-04	1.081977E-04
4	8.600335E-05	1.553609E-04	1.914267E-04	1.775551E-04	1.054234E-04
5	4.716213E-05	8.600335E-05	1.081977E-04	1.054234E-04	6.935733E-05

BELOW IS THE MASS MATRIX.

1	0.0023520
2	0.0023520
3	0.0023520
4	0.0023520
5	0.0023520

1	7.06E-03	-4.12E-10	4.66E-10	3.94E-11
2	-3.75E-10	9.41E-03	-2.46E-10	3.96E-10
3	4.72E-10	-2.27E-10	7.06E-03	-6.38E-10
4	5.59E-11	3.85E-10	-6.41E-10	9.41E-03

ABOVE IS THE MATRIX V TRANSPOSE M V , WHERE V IS THE MATRIX OF MODE SHAPES.
 THE MAXIMUM AND MINIMUM DIAGONAL VALUES ARE 0.009 AND 0.007
 THE MAXIMUM OFF DIAGONAL MAGNITUDE = 0.00000000

BELOW IS THE INERTIA MATRIX.

1	0.0070560
2	0.0094080
3	0.0070560
4	0.0094080

BELOW ARE THE SHAPES FOR MCODES

1	C.50C0000	1	-0.9999999	1	-0.9999995	1	1.0000000
2	C.8660254	2	-1.0000000	2	-0.0000002	2	-0.9999999
3	1.00C0000	3	-0.0000001	3	1.0000000	3	0.0000002
4	0.8660254	4	1.0000000	4	-0.0000000	4	0.9999993
5	0.50C0000	5	0.9999999	5	-0.9999997	5	-0.9999992

TABLE OF FINAL RESULTS

MODE NUMBER	RADIANS PER SECOND	FREQUENCY IN CPS	PERCENTAGE ACCURACY OF FREQUENCY	MODE NUMBER
1	38.7936053	6.1741940	100.000	1
2	155.0169086	24.6717072	100.000	2
3	346.6309357	55.1680264	100.000	3
4	600.3878544	95.5550127	100.000	4

PHASE II OUTPUT DATA -- OPTION 3, JOINT MOMENTS

RANDOM VIBRATION ANALYSIS SYSTEM FOR COMPLEX STRUCTURES (R A N V I B)

OPTION CONTROLS FLAG 1= 3 FLAG 2= 1 FLAG 3= 1 FLAG 4= 1 NPLATE= 0 NBEAMS= 6

NATURAL FREQUENCIES(RADIANS/SEC) GENERALIZED MASSES

1	0.3879360E 02	0.7056000E-02
2	0.1550169E 03	0.9407999E-02
3	0.3466309E 03	0.7055997E-02
4	0.6003898E 03	0.9407993E-02

LAMBDA= -0.000000 MU= -0.000000 G= 0.010000 N= 5 K= -0 NF= -0

RANDOM LOADING MODULE OUTPUT

FORCE CROSS POWER SPECTRAL DENSITY FOR DECAYED PROGRESSIVE WAVES FOR USE WITH PANEL -
SIMPLY SUPPORTED BEAM, OPTION 3 JOINT MOMENTS
81

I N P U T D A T A

THE FOLLOWING FOUR OPTIONS HAVE BEEN SELECTED (THEY APPEAR IN THE ORDER IN WHICH THEY WERE CARD INPUT) -

OPTION(S) (1) = 2 (2) = 2 (3) = 1 (4) = 1

PARAMETER VALUES ARE - D = 0.11890000E-02
CX = 0.72000000E 04
CY = -0.00000000E-38

NUMBER OF COORDINATES IN THE DIRECTION OF CYCLIC NODE NUMBERING - 3
NUMBER OF COORDINATES IN THE DIRECTION PERPENDICULAR TO THE CYCLIC NODE NUMBERING DIRECTION - 7

ORIGIN-TO-NODE LINE DISTANCES IN THE CYCLIC DIRECTION ARE -

0.0000000E-38 0.1111110E 00 0.2222220E 00

ORIGIN-TO-NODE LINE DISTANCES PERPENDICULAR TO THE CYCLIC DIRECTION ARE -

0.0000000E-38 0.9000000E 01 0.1800000E 02 0.2700000E 02 0.3600000E 02 0.4500000E 02
0.5400000E 02

AREA ASSOCIATED WITH EACH RETAINED NODE -

0.9999990E 00 0.9999990E 00 0.9999990E 00 0.9999990E 00 0.9999990E 00

PRESSURE POWER SPECTRAL DENSITIES ARE -

0.1000000E 01 0.1000000E 01 0.1000000E 01 0.1000000E 01

*****COMPUTED CONSTANTS*****

TRACE VELOCITY OF PRESSURE WAVE, CT = 0.7200000E 04
ANGLE BETWEEN TRACE WAVE PROPAGATION DIRECTION AND X-AXIS, THETA = 0.0000000E-38

THE FOLLOWING FORCE CROSS POWER SPECTRAL DENSITY MATRICES WERE GENERATED -

FREQUENCY NO.	1	CF(I,J) MATRIX	OMEGA =	0.38793605E 02
1	9.9999800E-01	9.8819108E-01	9.7422368E-01	9.5817564E-01
2	9.8819108E-01	9.9999800E-01	9.8819108E-01	9.7422368E-01
3	9.7422368E-01	9.8819108E-01	9.9999800E-01	9.8819108E-01
4	9.5817564E-01	9.7422368E-01	9.8819108E-01	9.9999800E-01
5	9.4013015E-01	9.5817564E-01	9.7422368E-01	9.8819108E-01

**** GENERATION COMPLETE ****

ADMITTANCE INTEGRALS

1	0.5404087E 02
2	0.4764180E 00
3	0.7575324E-01
4	0.8200228E-02

DEFLECTION CO-VARIANCE MATRIX (REAL)

1	1.834551E 02	3.206768E 02	3.694722E 02	3.192882E 02	1.840524E 02
2	3.206768E 02	5.549273E 02	6.399650E 02	5.535246E 02	3.192882E 02
3	3.694722E 02	6.399650E 02	7.389797E 02	6.399650E 02	3.694722E 02
4	3.192882E 02	5.535246E 02	6.399650E 02	5.549272E 02	3.206767E 02
5	1.840524E 02	3.192882E 02	3.694722E 02	3.206767E 02	1.854550E 02

REAL PART

103

```

5  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
6  0.00000E-38  0.37296E 06  0.00000E-38  0.00000E-38  0.00000E-38  0.18637E 05

```

```

BEAM 3  END 1

```

```

1  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
2  0.00000E-38  0.80373E 07  0.00000E-38  0.00000E-38  0.00000E-38  0.11205E 06
3  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
4  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
5  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
6  0.00000E-38  0.11205E 06  0.00000E-38  0.00000E-38  0.00000E-38  0.50629E 04

```

```

END 2

```

```

1  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
2  0.00000E-38  0.10464E 08  0.00000E-38  0.00000E-38  0.00000E-38  0.15767E 06
3  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
4  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
5  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
6  0.00000E-38  0.15762E 06  0.00000E-38  0.00000E-38  0.00000E-38  0.50629E 04

```

```

BEAM 4  END 1

```

```

1  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
2  0.00000E-38  0.10464E 08  0.00000E-38  0.00000E-38  0.00000E-38  -0.15762E 06
3  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
4  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
5  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
6  0.00000E-38  -0.15762E 06  0.00000E-38  0.00000E-38  0.00000E-38  0.50623E 04

```

```

END 2

```

```

1  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
2  0.00000E-38  0.80373E 07  0.00000E-38  0.00000E-38  0.00000E-38  -0.11206E 06
3  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
4  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
5  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38

```

105

DEFLECTION SECOND SPECTRAL MOMENT MATRIX (REAL PART)

1	2.974764E 05	4.970559E 05	5.546387E 05	4.660554E 05	2.614039E 05
2	4.970559E 05	8.521151E 05	9.631114E 05	8.160427E 05	4.660554E 05
3	5.546387E 05	9.631114E 05	1.113519E 06	9.631113E 05	5.546387E 05
4	4.660554E 05	8.160427E 05	9.631113E 05	8.521150E 05	4.970558E 05
5	2.614039E 05	4.660554E 05	5.546387E 05	4.970558E 05	2.974763E 05

REAL PART

107

```

4  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
5  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
6  0.00000E-38  0.23765E 10  0.00000E-38  0.00000E-38  0.00000E-38  0.45931E 09

```

```

BFAM 3  END 1

```

```

1  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
2  0.00000E-38  0.24331E 11  0.00000E-38  0.00000E-38  0.00000E-38  -0.11909E 10
3  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
4  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
5  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
6  0.00000E-38  -0.11909E 10  0.00000E-38  0.00000E-38  0.00000E-38  0.18655E 09

```

```

END 2

```

```

1  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
2  0.00000E-38  0.18006E 11  0.00000E-38  0.00000E-38  0.00000E-38  0.48806E 09
3  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
4  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
5  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
6  0.00000E-38  0.48807E 09  0.00000E-38  0.00000E-38  0.00000E-38  0.18655E 09

```

```

BFAM 4  END 1

```

```

1  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
2  0.00000E-38  0.18006E 11  0.00000E-38  0.00000E-38  0.00000E-38  -0.48806E 09
3  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
4  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
5  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
6  0.00000E-38  -0.48806E 09  0.00000E-38  0.00000E-38  0.00000E-38  0.18655E 09

```

```

END 2

```

```

1  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
2  0.00000E-38  0.24331E 11  0.00000E-38  0.00000E-38  0.00000E-38  0.11909E 10
3  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38
4  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38  0.00000E-38

```


6 0.00000E-38 0.13968E 03 0.00000E-38 0.00000E-38 0.00000E-38 0.23158E 09

R A N V I B P R O G R A M I S C O M P L E T E D

PHASE II OUTPUT DATA - OPTION 2, JOINT MOMENTS

RANDOM VIBRATION ANALYSIS SYSTEM FOR COMPLEX STRUCTURES (R A N V I B)

OPTION CONTROLS FLAG 1= 2 FLAG 2= 1 FLAG 3= 1 FLAG 4= 1 NPLATE= 0 NBEAMS= 6

NATURAL FREQUENCIES(RADIANS/SEC) GENERALIZED MASSES

1	0.3879360E 02	0.7C56000E-02
2	0.1550169E 03	0.9407999E-02
3	0.3466309E 03	0.7055997E-02
4	0.6003858E 03	0.9407993E-02

LAMBDA= -0.000000 MU= -0.000000 G= 0.010000 N= 5 K= 1 NF= -0

RANDOM LOADING MODULE OUTPUT

FORCE CROSS POWER SPECTRAL DENSITY FOR DECAYED PROGRESSIVE WAVES FOR USE WITH PANEL -
SIMPLY SUPPORTED BEAM, OPTION 2 JOINT MOMENTS
B1

I N P U T D A T A

THE FOLLOWING FOUR OPTIONS HAVE BEEN SELECTED (THEY APPEAR IN THE ORDER IN WHICH THEY WERE CARD INPUT) -

OPTION(S) (1) = 2 (2) = 2 (3) = 1 (4) = 1

PARAMETER VALUES ARE - D = 0.11890000E-02
CX = 0.72000000E 04
CY = -0.00000000E-38

NUMBER OF COORDINATES IN THE DIRECTION OF CYCLIC NODE NUMBERING - 3
NUMBER OF COORDINATES IN THE DIRECTION PERPENDICULAR TO THE CYCLIC NODE NUMBERING DIRECTION - 7

ORIGIN-TO-NODE LINE DISTANCES IN THE CYCLIC DIRECTION ARE -

0.0000000E-38 0.1111110E 00 0.2222220E 00

ORIGIN-TO-NODE LINE DISTANCES PERPENDICULAR TO THE CYCLIC DIRECTION ARE -

0.0000000E-38 0.9000000E 01 0.1800000E 02 0.2700000E 02 0.3600000E 02 0.4500000E 02
0.5400000E 02

AREA ASSOCIATED WITH EACH RETAINED NODE -

0.9999990E 00 0.9999990E 00 0.9999990E 00 0.9999990E 00 0.9999990E 00

PRESSURE POWER SPECTRAL DENSITIES ARE -

0.1000000E 01 0.1000000E 01 0.1000000E 01 0.1000000E 01

*****COMPUTED CONSTANTS*****

TRACE VELOCITY OF PRESSURE WAVE, CT = 0.72000000E 04
ANGLE BETWEEN TRACE WAVE PROPAGATION DIRECTION AND X-AXIS, THETA = 0.00000000E-38

THE FOLLOWING FORCE CROSS POWER SPECTRAL DENSITY MATRICES WERE GENERATED -

FREQUENCY NO. 1 CF(I,J) MATRIX OMEGA = 0.38793605E 02

1	9.9999800E-01	9.8819108E-01	9.7422368E-01	9.5817564E-01	9.4013015E-01
2	9.8819108E-01	9.9999800E-01	9.8819108E-01	9.7422368E-01	9.5817564E-01
3	9.7422368E-01	9.8819108E-01	9.9999800E-01	9.8819108E-01	9.7422368E-01
4	9.5817564E-01	9.7422368E-01	9.8819108E-01	9.9999800E-01	9.8819108E-01
5	9.4013015E-01	9.5817564E-01	9.7422368E-01	9.8819108E-01	9.9999800E-01

PRESSURE POWER SPECTRAL DENSITIES ARE -

0.1000000E 01 0.1000000E 01 0.1000000E 01

***** THIS IS AN OPTION 2 SOLUTION - RESULTS OF OPTION 3 APPEAR ABOVE FOLLOWED BY OPTION 2 BELOW *****
THE FOLLOWING FORCE CROSS POWER SPECTRAL DENSITY MATRICES WERE GENERATED -

FREQUENCY NO.	1	CF(I,J)	MATRIX	OMEGA =	0.96905255E 02
1	9.9999800E-01	9.8210462E-01	9.5023938E-01	9.0516376E-01	8.4781614E-01
2	9.8210462E-01	9.9999800E-01	9.8210462E-01	9.5023938E-01	9.0516376E-01
3	9.5023938E-01	9.8210462E-01	9.9999800E-01	9.8210462E-01	9.5023938E-01
4	9.0516376E-01	9.5023938E-01	9.8210462E-01	9.9999800E-01	9.8210462E-01
5	8.4781614E-01	9.0516376E-01	9.5023938E-01	9.8210462E-01	9.9999800E-01

```

FREQUENCY NO.  1  QF(I,J) MATRIX      OMEGA =  0.96905255E 02

  1  0.000000E-39  1.1954915E-01  2.3482002E-01  3.4421884E-01  4.4627136E-01
  2 -1.1954915E-01  0.0000000E-39  1.1954915E-01  2.3482002E-01  3.4421884E-01
  3 -2.3482002E-01 -1.1954915E-01  0.0000000E-39  1.1954915E-01  2.3482002E-01
  4 -3.4421884E-01 -2.3482002E-01 -1.1954915E-01  0.0000000E-39  1.1954915E-01
  5 -4.4627136E-01 -3.4421884E-01 -2.3482002E-01 -1.1954915E-01  0.0000000E-39

```

**** GENERATION COMPLETE ****

ADMITTANCE INTEGRALS (NO CROSS TERMS)

1	0.5404087E 02
2	0.4764180E 00
3	0.7575324E-01
4	0.8200228E-02

ADMITTANCE INTEGRALS

$$D(I)*D(J) + E(I)*E(J)$$

$$D(I)*E(J)$$

I= 1
 1 0.000000E-38
 2 -0.3451658E-02
 3 -0.3943656E-03
 4 -0.5681497E-04
 0.5404254E 02
 0.1807636E-03
 0.1727607E-04
 0.2347563E-05

I= 2
 1 0.1355335E-01
 2 0.000000E-38
 3 -0.3592549E-03
 4 -0.4473556E-04
 0.1807636E-03
 0.4764271E 00
 0.2569077E-04
 0.2368682E-05

I= 3
 1 0.3428542E-02
 2 0.7959793E-03
 3 0.000000E-38
 4 -0.8282542E-04
 0.1727607E-04
 0.2569077E-04
 0.7576157E-01
 0.7758042E-05

I= 4
 1 0.8497533E-03
 2 0.1703497E-03
 3 0.1424946E-03
 4 0.000000E-38
 0.2347563E-05
 0.2368682E-05
 0.7758042E-05
 0.8200995E-02

DEFLECTION COVARIANCE MATRIX (REAL PART)

1	1.854967E 02	3.207358E 02	3.695175E 02	3.193055E 02	1.840524E 02
2	3.207358E 02	5.550036E 02	6.400102E 02	5.535246E 02	3.192709E 02
3	3.695175E 02	6.400102E 02	7.389797E 02	6.399198E 02	3.694269E 02
4	3.193055E 02	5.535246E 02	6.399198E 02	5.548509E 02	3.206177E 02
5	1.840524E 02	3.192709E 02	3.694269E 02	3.206177E 02	1.854134E 02

DEFLECTION COVARIANCE MATRIX (IMAGINARY PART)

1	0.000000E-39	1.439736E-04	4.432555E-04	6.668238E-04	5.178314E-04
2	-1.439736E-04	0.000000E-39	4.407461E-04	8.107975E-04	6.668238E-04
3	-4.432555E-04	-4.407461E-04	0.000000E-39	4.407462E-04	4.432555E-04
4	-6.668238E-04	-8.107975E-04	-4.407462E-04	0.000000E-39	1.439736E-04
5	-5.178314E-04	-6.668238E-04	-4.432555E-04	-1.439736E-04	0.000000E-39

REAL PART

121

BEAM 5 END 1

END 2

BEAM 6 END 1

END 2

123

[illegible]

BEAM 3
END 1

END 2

BEAM 4 END 1

END 2

0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38
0.00000E+38	0.69479E+06	0.00000E+38	0.00000E+38	0.00000E+38	0.81771E 00
0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38
0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38

6 0.00000E-38 -0.17009E-06 0.00000E-38 0.00000E-38 0.00000E-38 0.16501E-07

DEFLECTION SECOND SPECTRAL MOMENT MATRIX (REAL PART)

1	2.975998E 05	4.973622E 05	5.549743E 05	4.662079E 05	2.614039E 05
2	4.973622E 05	8.525739E 05	9.634317E 05	8.160427E 05	4.659029E 05
3	5.549743E 05	9.634317E 05	1.113519E 06	9.627909E 05	5.543031E 05
4	4.662079E 05	8.160427E 05	9.627909E 05	8.516563E 05	4.967496E 05
5	2.614039E 05	4.659029E 05	5.543031E 05	4.967496E 05	2.973528E 05

DEFLECTION SECOND SPECTRAL MOMENT MATRIX (IMAGINARY PART)

1	0.000000E-39	-6.338922E-01	1.673028E 00	5.509748E 00	5.099172E 00
2	6.338922E-01	0.000000E-39	1.150794E 00	4.875857E 00	5.509749E 00
3	-1.673028E 00	-1.150794E 00	0.000000E-39	1.150795E 00	1.673029E 00
4	-5.509748E 00	-4.875857E 00	-1.150795E 00	0.000000E-39	-6.338925E-01
5	-5.099172E 00	-5.509749E 00	-1.673029E 00	6.338925E-01	0.000000E-39

BEAM 1 1
END 1

END 2

BEAM 2 END 1

END 2

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.24359E 11	0.00000E-38	0.23790E 10
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38

BEAM 5 END 1

5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.11914E 10	0.00000E-38	0.00000E-38	0.18700E 09

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.24303E 11	0.00000E-38	0.00000E-38	0.00000E-38	-0.23740E 10
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.23740E 10	0.00000E-38	0.00000E-38	0.00000E-38	0.46952E 09

END 2

END 2

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.18873E 11	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.17706E 10
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.17706E 10	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.46052E 09

REAM

BEAM 6
END 1

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.16873E 11	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.20970E 10
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.20970E 10	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.23300E 09

END 2

END 2

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.19732E-03	0.00000E-38	0.00000E-38	0.13993E 03
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38

5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-36	0.13993E 03	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.23300E 09

IMAGINARY PART

BEAM 1	END 1								
1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.28919E-19	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.63094E-04
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.63094E-04	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.12798E-03

END 2									
1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	-0.82299E-04	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.81468E-02
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.65859E-02	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.12798E-03

BEAM 2	END 1								
1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	-0.44875E-02	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.18777E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.18777E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.65568E-04

END 2									
1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	-0.59393E-02	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.18777E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38

135

6 0.00000E-38 0.33246E-02 0.00000E-38 0.00000E-38 0.00000E-38 -0.67348E-04

RANVIB PROGRAM IS COMPLETED

PHASE 1 | INPUT DATA -- OPTION 3, CROSS PSD

[illegible]

PHASE II OUTPUT DATA -- OPTION 3, CROSS PSD

RANDOM VIBRATION ANALYSIS SYSTEM FOR COMPLEX STRUCTURES (R A N V I B)

OPTION CONTROLS FLAG 1= 3 FLAG 2= 2 FLAG 3= 1 FLAG 4=0 NPLATE= -0 NBEAMS= 6

NATURAL FREQUENCIES(RADIANS/SEC) GENERALIZED MASSES

1	0.3879360E 02	0.7056000E-02
2	0.1550169E 03	0.9407999E-02
3	0.3466309E 03	0.7055997E-02
4	0.6003898E 03	0.9407993E-02

LAMBDA= -0.000000 MU= G= -0.000000 G= 0.010000 N= 5 K= -0 NF= 1

CROSS-PSD FREQUENCIES (RAD/SEC)

1 0.3879000E 02

RANDOM LOADING MODULE OUTPUT

FORCE CROSS POWER SPECTRAL DENSITY FOR DECAYED PROGRESSIVE WAVES FOR USE WITH PANEL -
SIMPLY SUPPORTED BEAM, OPTION 3 CROSS PSD

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INPUT DATA

THE FOLLOWING FOUR OPTIONS HAVE BEEN SELECTED (THEY APPEAR IN THE ORDER IN WHICH THEY WERE CARD INPUT) -

OPTION(S) (1) = 2 (2) = 2 (3) = 1 (4) = 1

PARAMETER VALUES ARE - D = 0.11890000E-02
CX = 0.72000000E 04
CY = -0.00000000E-38

NUMBER OF COORDINATES IN THE DIRECTION OF CYCLIC NODE NUMBERING - 3
NUMBER OF COORDINATES IN THE DIRECTION PERPENDICULAR TO THE CYCLIC NODE NUMBERING DIRECTION - 7

ORIGIN-TO-NODE LINE DISTANCES IN THE CYCLIC DIRECTION ARE -

0.00000000E-38 0.1111110E 00 0.2222220E 00

ORIGIN-TO-NODE LINE DISTANCES PERPENDICULAR TO THE CYCLIC DIRECTION ARE -

0.00000000E-38 0.9000000E 01 0.1800000E 02 0.2700000E 02 0.3600000E 02 0.4500000E 02
0.5400000E 02

AREA ASSOCIATED WITH EACH RETAINED NODE -

0.9999999E 00 0.9999999E 00 0.9999999E 00 0.9999999E 00 0.9999999E 00

PRESSURE POWER SPECTRAL DENSITIES ARE -

0.1000000E 01

*****COMPUTED CONSTANTS*****

TRACE VELOCITY OF PRESSURE WAVE, CT = 0.72000000E 04
ANGLE BETWEEN TRACE WAVE PROPAGATION DIRECTION AND X-AXIS, THETA = 0.00000000E-38

THE FOLLOWING FORCE CROSS POWER SPECTRAL DENSITY MATRICES WERE GENERATED -

FREQUENCY NO.	1	CF(I,J) MATRIX	OMEGA =	0.38790000E 02
1	9.9999800E-01	9.8819129E-01	9.7422455E-01	9.5817754E-01
2	9.8819129E-01	9.9999800E-01	9.8819129E-01	9.7422455E-01
3	9.7422455E-01	9.8819129E-01	9.9999800E-01	9.8819129E-01
4	9.5817754E-01	9.7422455E-01	9.8819129E-01	9.9999800E-01
5	9.4013348E-01	9.5817754E-01	9.7422455E-01	9.8819129E-01

*** GENERATION COMPLETE ***

ADMITTANCE SCALARS (D**2+E**2)

FREQUENCY= 0.387900E 02(RAD/SEC)
1 0.8865278E 02
2 0.2226405E-04
3 0.1426643E-05
4 0.8767225E-07

BELOW ARE DEFLECTION CROSS-PSD MATRICES

DEFLECTION CO-POWER, FREQUENCY=		38.790000 (RAD/SEC)			
1	3.030651E 02	5.249242E 02	6.061302E 02	5.249241E 02	3.030651E 02
2	5.249242E 02	9.091953E 02	1.049848E 03	9.091953E 02	5.249241E 02
3	6.061302E 02	1.049848E 03	1.212260E 03	1.049848E 03	6.061301E 02
4	5.249241E 02	9.091953E 02	1.049848E 03	9.091952E 02	5.249241E 02
5	3.030651E 02	5.249241E 02	6.061301E 02	5.249241E 02	3.030651E 02

BEAM	1	END	1			
	1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	2	0.00000E-38	0.61214E-08	0.00000E-38	0.00000E-38	-0.17993E-01
	3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	6	0.00000E-38	-0.17993E-01	0.00000E-38	0.00000E-38	0.52887E 05
	END	2				
	1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	2	0.00000E-38	0.42838E 07	0.00000E-38	0.00000E-38	0.47598E 06
	3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	6	0.00000E-38	0.47598E 06	0.00000E-38	0.00000E-38	0.52887E 05
BEAM	2	END	1			
	1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	2	0.00000E-38	0.42838E 07	0.00000E-38	0.00000E-38	0.34843E 06
	3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	6	0.00000E-38	0.34843E 06	0.00000E-38	0.00000E-38	0.28344E 05
	END	2				
	1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	2	0.00000E-38	0.12851E 08	0.00000E-38	0.00000E-38	0.60353E 06
	3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38

BEAM 5 END 1

6	0.00000E-38	-0.22089E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.38001E 04
1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.12851E 08	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.60353E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.60353E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.28344E 05

END 2

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.42838E 07	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.34843E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.34843E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.28344E 05

BEAM 6 END 1

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.42838E 07	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.47598E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.47598E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.52887E 05

END 2

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.19505E-06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.10156E 00
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.10156E 00	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.52887E 05

RANVIB PROGRAM IS COMPLETED

[illegible]

PHASE II OUTPUT DATA -- OPTION 2, CROSS PSD

RANDOM VIBRATION ANALYSIS SYSTEM FOR COMPLEX STRUCTURES (R A N V I B)

OPTION CONTROLS FLAG 1= 2 FLAG 2= 2 FLAG 3= 1 FLAG 4=-0 NPLATE= -0 NBEAMS= 6

NATURAL FREQUENCIES(RADIANS/SEC) GENERALIZED MASSES

1	0.3879360E 02	0.7056000E-02
2	0.1550169E 03	0.9407999E-02
3	0.3466309E 03	0.7055997E-02
4	0.6003898E 03	0.9407993E-02

LAMBDA= -0.000000 MU= -0.000000 G= 0.010000 N= 5 K= 1 NF= 1

CROSS-PSD FREQUENCIES(RAD/SEC)

1 0.3879000E 02

RANDOM LOADING MODULE OUTPUT

FORCE CROSS POWER SPECTRAL DENSITY FOR DECAYED PROGRESSIVE WAVES FOR USE WITH PANEL -
SIMPLY SUPPORTED BEAM, OPTION 2 CROSS PSD

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I N P U T D A T A

THE FOLLOWING FOUR OPTIONS HAVE BEEN SELECTED (THEY APPEAR IN THE ORDER IN WHICH THEY WERE CARD INPUT) -

OPTION(S) (1) = 2 (2) = 2 (3) = 0 (4) = 1

PARAMETER VALUES ARE - D = 0.11890000E-02
CX = 0.72000000E 04
CY = -0.00000000E-38

NUMBER OF COORDINATES IN THE DIRECTION OF CYCLIC NODE NUMBERING - 3
NUMBER OF COORDINATES IN THE DIRECTION PERPENDICULAR TO THE CYCLIC NODE NUMBERING DIRECTION - 7

ORIGIN-TO-NODE LINE DISTANCES IN THE CYCLIC DIRECTION ARE -

0.0000000E-38 0.1111110E 00 0.2222220E 00

ORIGIN-TO-NODE LINE DISTANCES PERPENDICULAR TO THE CYCLIC DIRECTION ARE -

0.0000000E-38 0.9000000E 01 0.1800000E 02 0.2700000E 02 0.3600000E 02 0.4500000E 02
0.5400000E 02

AREA ASSOCIATED WITH EACH RETAINED NODE -

0.9999990E 00 0.9999990E 00 0.9999990E 00 0.9999990E 00 0.9999990E 00

PRESSURE POWER SPECTRAL DENSITIES ARE -

0.1000000E 01

*****COMPUTED CONSTANTS*****

TRACE VELOCITY OF PRESSURE WAVE, CT = 0.72000000E 04
ANGLE BETWEEN TRACE WAVE PROPAGATION DIRECTION AND X-AXIS, THETA = 0.00000000E-38

THE FOLLOWING FORCE CROSS POWER SPECTRAL DENSITY MATRICES WERE GENERATED -

**** GENERATION COMPLETE ****

ADMITTANCE SCALARS (D**2+E**2)

FREQUENCY= 0.3879000E 02(RAD/SEC)
1 0.8865278E 02
2 0.2226405E-04
3 0.1426643E-05
4 0.8767225E-07

ADMITTANCE SCALARS

$$D(I)*D(J) + E(I)*E(J)$$

$$D(I)*E(J)$$

I= 1 1 0.1646952E 01
 2 0.8805811E-05
 3 0.2116012E-05
 4 0.5201584E-06
 0.8865278E 02
 0.1299282E-02
 0.3228136E-03
 0.7978833E-04

I= 2 1 0.4441694E-01
 2 0.2374854E-06
 3 0.5706710E-07
 4 0.1402824E-07
 0.1299282E-02
 0.2226405E-04
 0.5635854E-05
 0.1397118E-05

I= 3 1 0.1124363E-01
 2 0.6011669E-07
 3 0.1444588E-07
 4 0.3551087E-08
 0.3228136E-03
 0.5635854E-05
 0.1426643E-05
 0.3536623E-06

I= 4 1 0.2787279E-02
 2 0.1490283E-07
 3 0.3581110E-08
 4 0.8803087E-09
 0.7978833E-04
 0.1397118E-05
 0.3536623E-06
 0.8767225E-07

DEFLECTION CROSS PSD MATRICES

DEFLECTION CO-POWER, FREQUENCY = 0.3879000E 02 (RAD/SEC)

1	3.031119E 02	5.249880E 02	6.061770E 02	5.249412E 02	3.030651E 02
2	5.249880E 02	9.092763E 02	1.049895E 03	9.091953E 02	5.249070E 02
3	6.061770E 02	1.049895E 03	1.212260E 03	1.049801E 03	6.060834E 02
4	5.249412E 02	9.091953E 02	1.049801E 03	9.091143E 02	5.248602E 02
5	3.030651E 02	5.249070E 02	6.060834E 02	5.248602E 02	3.030183E 02

DEFLECTION QUAD-POWER, FREQUENCY = 0.3879000E 02 (RAD/SEC)

1	0.000000E-39	4.992252E-04	1.366412E-03	1.870148E-03	1.371051E-03
2	-4.992252E-04	0.000000E-39	1.366476E-03	2.369374E-03	1.870148E-03
3	-1.366412E-03	-1.366476E-03	0.000000E-39	1.366476E-03	1.366412E-03
4	-1.870148E-03	-2.369374E-03	-1.366476E-03	0.000000E-39	4.992253E-04
5	-1.371051E-03	-1.870148E-03	-1.366412E-03	-4.992253E-04	0.000000E-39

STRESS CROSS-PSD MATRICES

BEAM 1

END 1

REAL PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.61217E-08	0.00000E-38	0.00000E-38	0.00000E-38	-0.17999E-01
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.17999E-01	0.00000E-38	0.00000E-38	0.00000E-38	0.52922E 05

IMAG PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	-0.10507E-23	0.00000E-38	0.00000E-38	0.00000E-38	0.16285E-06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.16285E-06	0.00000E-38	0.00000E-38	0.00000E-38	0.23900E-07

END 2

REAL PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.42867E 07	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.47630E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.47630E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.52922E 05

IMAG PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	-0.37711E-06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.27083E-05
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.25351E-05	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.23900E-07

BEAM 2

END 1

REAL PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.42867E 07	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.34855E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.34855E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.28344E 05

IMAG PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	-0.48221E-05	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.32673E 01
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.32673E 01	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.46217E-08

END 2

REAL PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.12856E 08	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.60364E 06

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.40419E-05	0.00000E-38	0.00000E-38	0.00000E-38	0.32673E 01	
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	
6	0.00000E-38	-0.32673E 01	0.00000E-38	0.00000E-38	0.00000E-38	-0.46217E-08	

BEAM 3

END 1

REAL PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.12856E-08	0.00000E-38	0.00000E-38	0.00000E-38	0.22066E-06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38

5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.2206E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.37913E 04

IMAG PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.36573E-05	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.90948E 01
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.90948E 01	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.88172E-07

END 2

REAL PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.17135E 08	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.25478E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.25478E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.37913E 04

IMAG PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.19887E-06	0.00000E-38	0.00000E-38	0.00000E-38	0.90948E 01
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.90948E 01	0.00000E-38	0.00000E-38	0.00000E-38	0.88172E-07

BEAM 4

END 1

REAL PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.17135E 08	0.00000E-38	0.00000E-38	0.00000E-38	-0.25541E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.25541E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.38115E 04

IMAG PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	-0.47137E-06	0.00000E-38	0.00000E-38	0.00000E-38	0.90948E 01
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.90948E 01	0.00000E-38	0.00000E-38	0.00000E-38	0.21288E-06

END 2

REAL PART

FREQUENCY = 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.12847E 08	0.00000E-38	0.00000E-38	0.00000E-38	-0.22111E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.22111E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.38115E 04

IMAG PART

FREQUENCY = 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.34048E-05	0.00000E-38	0.00000E-38	0.00000E-38	0.90948E 01
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38

END 1

REAL PART

FREQUENCY= 0.3879000E 02

4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.30948E 01	0.00000E-38	0.00000E-38	0.21288E-06

IMAG PART

FREQUENCY = 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.80118E-06	0.00000E-38	0.00000E-38	0.00000E-38	0.32672E 01
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38

END 2

REAL PART

FREQUENCY= 0.3879000E 02

6	0.00000E-38	-0.32672E 01	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.19540E-08
1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.42810E 07	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.34831E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.34831E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.28345E 05

IMAG PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	-0.15853E-05	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.32672E 01
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.32672E 01	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.19540E-08

BEAM 6

END 1

REAL PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.42810E 07	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.47567E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.47567E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.52852E 05

IMAG PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	-0.58465E-07	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.32640E-05
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.35258E-05	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.29812E-07

END 2

REAL PART

FREQUENCY= 0.3879000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.19501E-06	0.00000E-38	0.00000E-38	0.00000E-38	0.10152E 00
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.10152E 00	0.00000E-38	0.00000E-38	0.00000E-38	0.52852E 05

IMAG PART

FREQUENCY= 0.3679000E 02

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.79119E-21	0.00000E-38	0.00000E-38	0.00000E-38	0.70026E-06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.70026E-06	0.00000E-38	0.00000E-38	0.00000E-38	0.29812E-07

RANVIB PROGRAM IS COMPLETED

[illegible]

PHASE II OUTPUT DATA -- OPTION 1, CROSS PSD

RANDOM VIBRATION ANALYSIS SYSTEM FOR COMPLEX STRUCTURES (R A N V I B)

OPTION CONTROLS FLAG 1= 1 FLAG 2= 2 FLAG 3= 1 FLAG 4=-0 NPLATE= -0 NBEAMS= 6

NATURAL FREQUENCIES(RADIANS/SEC) GENERALIZED MASSES

1	0.3879360E 02	0.7056000E-02
2	0.1550169E 03	0.9407999E-02
3	0.3466309E 03	0.7055997E-02
4	0.6003898E 03	0.9407993E-02

LAMBDA= -0.000000 MU= -0.000000 G= -0.000000 N= 5 K= -0 NF= 1

CROSS-PSD FREQUENCIES (RAD/SEC)

1 0.3879000E 02

RANDOM LOADING MODULE OUTPUT

FORCE CROSS POWER SPECTRAL DENSITY FOR DECAYED PROGRESSIVE WAVES FOR USE WITH PANEL -
SIMPLY SUPPORTED BEAM, OPTION 1 CROSS PSD

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I N P U T D A T A

THE FOLLOWING FOUR OPTIONS HAVE BEEN SELECTED (THEY APPEAR IN THE ORDER IN WHICH THEY WERE CARD INPUT) -

OPTION(S) (1) = 2 (2) = 2 (3) = -0 (4) = 1

PARAMETER VALUES ARE - D = 0.11890000E-02
CX = 0.72000000E 04
CY = -0.00000000E-38

NUMBER OF COORDINATES IN THE DIRECTION OF CYCLIC NODE NUMBERING - 3
NUMBER OF COORDINATES IN THE DIRECTION PERPENDICULAR TO THE CYCLIC NODE NUMBERING DIRECTION - 7

ORIGIN-TO-NODE LINE DISTANCES IN THE CYCLIC DIRECTION ARE -

0.00000000E-38 0.1111110E 00 0.2222220E 00

ORIGIN-TO-NODE LINE DISTANCES PERPENDICULAR TO THE CYCLIC DIRECTION ARE -

0.00000000E-38 0.9000000E 01 0.1800000E 02 0.2700000E 02 0.3600000E 02 0.4500000E 02
0.5400000E 02

AREA ASSOCIATED WITH EACH RETAINED NODE -

0.9999990E 00 0.9999990E 00 0.9999990E 00 0.9999990E 00 0.9999990E 00

PRESSURE POWER SPECTRAL DENSITIES ARE -

0.1000000E 01

*****COMPUTED CONSTANTS*****

TRACE VELOCITY OF PRESSURE WAVE, CT = 0.72000000E 04
ANGLE BETWEEN TRACE WAVE PROPAGATION DIRECTION AND X-AXIS, THETA = 0.00000000E-38

THE FOLLOWING FORCE CROSS POWER SPECTRAL DENSITY MATRICES WERE GENERATED -

**** GENERATION COMPLETE ****

DAMPING MATRIX

1	1.199208E-01	-1.154396E-01	5.043680E-02	-1.345100E-02	3.363000E-03
2	-1.154396E-01	1.703576E-01	-1.288905E-01	5.379980E-02	-1.345100E-02
3	5.043680E-02	-1.288905E-01	1.737206E-01	-1.288905E-01	5.043680E-02
4	-1.345100E-02	5.379980E-02	-1.288905E-01	1.703576E-01	-1.154396E-01
5	3.363000E-03	-1.345100E-02	5.043680E-02	-1.154396E-01	1.199208E-01

DEFLECTION CROSS PSD MATRICES

DEFLECTION CO-POWER, FREQ= 0.3879000E 02 (RAD/SEC)

1	3.072461E 02	5.321462E 02	6.144423E 02	5.321001E 02	3.071980E 02
2	5.321462E 02	9.216703E 02	1.064206E 03	9.215903E 02	5.320629E 02
3	6.144423E 02	1.064206E 03	1.228785E 03	1.064114E 03	6.143461E 02
4	5.321000E 02	9.215902E 02	1.064114E 03	9.215103E 02	5.320167E 02
5	3.071979E 02	5.320628E 02	6.143460E 02	5.320167E 02	3.071498E 02

DEFLECTION QUAD-POWER, FREQ= 0.3879000E 02 (RAD/SEC)

1	-1.035810E-03	-3.784561E-02	-6.139779E-02	-3.643799E-02	3.580451E-04
2	3.463176E-02	-2.112627E-03	-3.309441E-02	3.199577E-04	3.724039E-02
3	5.914104E-02	3.033400E-02	-3.705025E-04	3.313994E-02	6.191933E-02
4	3.577280E-02	-4.775524E-04	-3.120399E-02	1.954556E-03	3.818119E-02
5	-8.791685E-05	-3.619838E-02	-5.949306E-02	-3.479123E-02	1.305640E-03

STRESS CROSS-PSD MATRICES

BEAM 1

FREQUENCY= 0.3879000E 02
END 1

REAL PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.62051E-08	0.00000E-38	0.00000E-38	0.00000E-38	-0.18245E-01
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.18245E-01	0.00000E-38	0.00000E-38	0.00000E-38	0.53649E 05

IMAG PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	-0.69612E-14	0.00000E-38	0.00000E-38	0.00000E-38	-0.13695E-04
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.13906E-04	0.00000E-38	0.00000E-38	0.00000E-38	-0.55828E 00

END 2

REAL PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.43456E 07	0.00000E-38	0.00000E-38	0.00000E-38	0.48284E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.48284E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.53649E 05

IMAG PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	-0.45220E 02	0.00000E-38	0.00000E-38	0.00000E-38	-0.50247E 01
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.50242E 01	0.00000E-38	0.00000E-38	0.00000E-38	-0.55828E 00

BEAM 2

FREQUENCY= 0.3879000E 02
END 1

REAL PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
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1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	-0.45220E 02	0.00000E-38	0.00000E-38	0.00000E-38	-0.67244E 03
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.66523E 03	0.00000E-38	0.00000E-38	0.00000E-38	-0.25952E 00

REAL PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.13031E 08	0.00000E-38	0.00000E-38	0.61178E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38

6 0.00000E-38 0.61178E 06 0.00000E-38 0.00000E-38 0.00000E-38 0.28722E 05

IMAG PART

1 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38
 2 0.00000E-38 -0.13112E 03 0.00000E-38 0.00000E-38 0.00000E-38 -0.67478E 03
 3 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38
 4 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38
 5 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38
 6 0.00000E-38 0.66290E 03 0.00000E-38 0.00000E-38 0.00000E-38 -0.25952E 00

BEAM 3

FREQUENCY= 0.3879000E 02
 END 1

REAL PART

1 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38
 2 0.00000E-38 0.13031E 08 0.00000E-38 0.00000E-38 0.00000E-38 0.22369E 06
 3 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38
 4 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38
 5 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38 0.00000E-38
 6 0.00000E-38 0.22370E 06 0.00000E-38 0.00000E-38 0.00000E-38 0.38394E 04

IMAG PART

REAL PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.17368E 08	0.00000E-38	0.00000E-38	0.00000E-38	0.25825E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
	0.00000E-38	0.25825E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.38394E 04

IMAG PART

1	0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38
2	0.00000E+38	-0.36512E 02	0.00000E+38	0.00000E+38	0.00000E+38	-0.41402E 03
3	0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38	0.00000E+38

BEAM 4

FREQUENCY= 0.3879000E 02
END 1

4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.42661E 03	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.23081E 00

REAL PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.17368E 08	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.25882E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.25882E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.38561E 04

IMAG PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	-0.36512E 02	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.43310E 03
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.41397E 03	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.23993E 00

END 2

REAL PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.13022E 08	0.00000E-38	0.00000E-38	0.00000E-38	-0.22411F 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.22411E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.38561E 04

IMAG PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.11623E 03	0.00000E-38	0.00000E-38	0.00000E-38	0.43094F 03
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.41613E 03	0.00000E-38	0.00000E-38	0.00000E-38	-0.23993E 00

BEAM 5

FREQUENCY= 0.3879000E 02
END 1

REAL PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.13022E 08	0.00000E-38	0.00000E-38	0.00000E-38	-0.61168E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.61168E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.28732E 05

I'MAG PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.11623E 03	0.00000E-38	0.00000E-38	0.00000E-38	0.66920E 03
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.66931E 03	0.00000E-38	0.00000E-38	0.00000E-38	-0.24415E 00

END 2

REAL PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.43393E 07	0.00000E-38	0.00000E-38	0.00000E-38	-0.35309E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38

5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.35309E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.28732E 05

IMAG PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.95456E 02	0.00000E-38	0.00000E-38	0.00000E-38	0.66700E 03
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.67151E 03	0.00000E-38	0.00000E-38	0.00000E-38	-0.24415E 00

BEAM 6

FREQUENCY= 0.3879000E 02
END 1

REAL PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.43393E 07	0.00000E-38	0.00000E-38	0.00000E-38	-0.48214E 06
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.48214E 06	0.00000E-38	0.00000E-38	0.00000E-38	0.53571E 05

IMAG PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.95456E 02	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	-0.10606E 02
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.10607E 02	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.11785E 01

END 2

REAL PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.19767E-06	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.10290E 00
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	0.10290E 00	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.51571E 05

IMAG PART

1	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
2	0.00000E-38	0.10261E-11	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.69925E-04
3	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38

4	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
5	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38
6	0.00000E-38	-0.67129E-04	0.00000E-38	0.00000E-38	0.00000E-38	0.00000E-38	0.11785E 01

R A N V I B P R O G R A M I S C O M P L E T E D

PHASE I INPUT DATA -- SANDWICH PLATE

[illegible]

1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	73	75	77	79	
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	
	1		1		2				1							1		-1	.0																				D2	
																	-1	.0																					D3	
	2		2		3										1																								D4	
																	-1	.0																					D2	
	3		3		4										1																								D3	
																	-1	.0																					D2	
	4		4		5										1																								D3	
																	-1	.0																					D2	
	5		21		22										1																									D3
																	-1	.0																					D2	
	6		22		23										1																									D3
1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	73	75	77	79	
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	
																	-1	.0																					D3	
	7		23		24										1																									D2
																	-1	.0																					D3	
	8		24		25										1																									D2
																	-1	.0																					D3	
																																							E1	
				15					4																														E2	
	0.0006	53		53		0.0006	53		53		0.0006	53		53		0.0008	08		08		0.0008	08		08		0.0008	08		08		0.0008	08		08		0.0008	08		E2	
	0.0008	08		08		0.0008	08		08		0.0008	08		08		0.0008	08		08		0.0008	08		08		0.0008	08		08		0.0008	08		08		0.0008	08		E2	
	0.0006	53		53																																		E2		

PHASE I OUTPUT DATA -- SANDWICH PLATE

CALCULATE DEFLECTIONS ONLY

STRUCTURE SIZE	BEAMS	PLATES	NODES	PARTITIONS
	8	16	25	-C

LAST NODES IN PARTITIONS
1, 10 2, 20 3, 25

SANDWICH PLATE

NODE	RC	NODAL DATA			Z	RETAINED FREEDOMS
		X	Y			
1	11111	0.0000E-39	0.0000E-39		-0.0000E-39	000000
2	01110	6.0000E 00	0.0000E-39		-0.0000E-39	000001
3	01110	1.2000E 01	0.0000E-39		-0.0000E-39	000001
4	01110	1.8000E 01	0.0000E-39		-0.0000E-39	000001
5	11111	2.4000E 01	0.0000E-39		-0.0000E-39	000000
6	10111	0.0000E-39	7.5000E 00		-0.0000E-39	000000
7	00110	6.0000E 00	7.5000E 00		-0.0000E-39	000001
8	00110	1.2000E 01	7.5000E 00		-0.0000E-39	000001
9	00110	1.8000E 01	7.5000E 00		-0.0000E-39	000001
10	10111	2.4000E 01	7.5000E 00		-0.0000E-39	000000
11	10111	0.0000E-39	1.5000E 01		-0.0000E-39	000001
12	00110	6.0000E 00	1.5000E 01		-0.0000E-39	000001
13	00110	1.2000E 01	1.5000E 01		-0.0000E-39	000000
14	00110	1.8000E 01	1.5000E 01		-0.0000E-39	000001
15	10111	2.4000E 01	1.5000E 01		-0.0000E-39	000000
16	10111	0.0000E-39	2.2500E 01		-0.0000E-39	000001
17	00110	6.0000E 00	2.2500E 01		-0.0000E-39	000001
18	00110	1.2000E 01	2.2500E 01		-0.0000E-39	000001
19	00110	1.8000E 01	2.2500E 01		-0.0000E-39	000001
20	10111	2.4000E 01	2.2500E 01		-0.0000E-39	000000
21	11111	0.0000E-39	3.0000E 01		-0.0000E-39	000000
22	01110	6.0000E 00	3.0000E 01		-0.0000E-39	000001
23	01110	1.2000E 01	3.0000E 01		-0.0000E-39	000001
24	01110	1.8000E 01	3.0000E 01		-0.0000E-39	000001
25	11111	2.4000E 01	3.0000E 01		-0.0000E-39	000000

SANDWICH PLATE

PLATE DATA

PLATE	NODE	NODE	NODE	NODE	T(O)	T(X)	T(Y)	T(X)	T(Y)	T(S)	I(S)	E	GAMMA
	1	2	3	4									
1	1	2	6	7	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
2	2	3	7	8	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
3	3	4	8	9	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
4	4	5	9	10	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
5	5	6	7	11	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
6	6	7	8	12	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
7	7	8	9	13	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
8	8	9	10	14	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
9	9	11	12	15	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
10	10	12	13	16	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
11	11	13	14	17	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
12	12	14	15	18	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
13	13	16	17	21	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
14	14	17	18	22	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
15	15	18	19	23	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01
16	16	19	20	24	4.000E-02	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	4.000E-02	2.500E-03	1.050E 07	3.300E-01

SANDWICH PLATE

BEAM DATA

BEAM	NODE	NODE	NODE	FIXITY	I(Y)	A(XY)	A	I(Z)	A(XZ)	J	YG MOD	SH MOD
1	1	2	3									
OFFSETS	-0.0000E-39	-0.0000E-39	-0.0000E-39	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06
2	2	3	-0	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06
OFFSETS	-0.0000E-39	-0.0000E-39	-0.0000E-39	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06
3	3	4	-0	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06
OFFSETS	-0.0000E-39	-0.0000E-39	-0.0000E-39	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06
4	4	5	-0	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06
OFFSETS	-0.0000E-39	-0.0000E-39	-0.0000E-39	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06
5	21	22	-0	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06
OFFSETS	-0.0000E-39	-0.0000E-39	-0.0000E-39	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06
6	22	23	-0	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06
OFFSETS	-0.0000E-39	-0.0000E-39	-0.0000E-39	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06
7	23	24	-0	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06
OFFSETS	-0.0000E-39	-0.0000E-39	-0.0000E-39	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06
8	24	25	-0	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06
OFFSETS	-0.0000E-39	-0.0000E-39	-0.0000E-39	000000	1.000E-01	-0.000E-39	-0.000E-39	-0.000E-39	-0.000E-39	8.000E-05	1.050E 07	3.900E 06

BELOW IS THE COMPLETE FLEXIBILITY MATRIX UNSCALED. THE SCALE FACTOR = 1.000000E 00

1	8.872197E-06 9.567012E-07 1.792341E-09	5.767948E-06 5.759828E-07	2.866753E-06 2.979886E-07	2.165572E-06 3.769239E-07	2.182878E-06 2.377354E-07	1.280997E-06 2.944445E-09	8.513209E-07 3.169535E-09
2	5.767948E-06 1.427203E-06 3.169531E-09	1.178995E-05 9.567010E-07	5.767948E-06 3.769237E-07	2.182878E-06 5.356336E-07	3.446569E-06 3.769237E-07	2.182878E-06 3.169531E-09	9.567010E-07 4.737777E-09
3	2.866753E-06 9.567012E-07 2.944445E-09	5.767948E-06 8.513209E-07	8.872197E-06 2.377354E-07	1.280997E-06 3.769239E-07	2.182878E-06 2.979886E-07	2.165572E-06 1.792341E-09	5.759828E-07 3.169535E-09
4	2.165571E-06 1.092634E-04 2.377350E-07	2.182877E-06 6.380581E-05	1.280996E-06 4.111963E-05	1.081804E-04 5.460216E-05	1.078926E-04 3.590069E-05	5.579536E-05 2.978981E-07	9.456629E-05 3.769234E-07
5	2.182877E-06 1.583721E-04 3.769234E-07	3.446568E-06 1.092634E-04	2.182877E-06 5.460216E-05	1.078926E-04 7.702033E-05	1.639758E-04 5.460216E-05	1.078926E-04 3.769234E-07	1.092634E-04 5.356336E-07
6	1.280996E-06 1.092634E-04 2.978981E-07	2.182877E-06 9.456629E-05	2.165571E-06 3.590069E-05	5.579536E-05 5.460216E-05	1.078926E-04 4.111963E-05	1.081804E-04 2.377350E-07	6.380581E-05 3.769234E-07
7	8.513202E-07 1.617829E-04 5.759822E-07	9.567006E-07 9.123079E-05	5.759823E-07 9.456631E-05	9.456629E-05 1.092634E-04	1.092634E-04 6.380582E-05	6.380581E-05 8.513202E-07	1.487273E-04 9.567004E-07
8	9.567004E-07 2.309581E-04 9.567004E-07	1.427203E-06 1.617829E-04	9.567004E-07 1.092634E-04	1.092634E-04 1.583721E-04	1.583721E-04 1.092634E-04	1.092634E-04 9.567004E-07	1.617829E-04 1.427203E-06
9	5.759823E-07 1.617829E-04 8.513202E-07	9.567006E-07 1.487273E-04	8.513202E-07 6.380582E-05	6.380581E-05 1.092634E-04	1.092634E-04 9.456631E-05	9.456629E-05 5.759822E-07	9.123079E-05 9.567004E-07
10	2.978993E-07 1.092634E-04 1.280996E-06	3.769237E-07 6.380583E-05	2.377352E-07 1.081804E-04	4.111964E-05 1.078926E-04	5.460217E-05 5.579538E-05	2.590070E-05 2.165572E-06	9.456631E-05 2.182878E-06
11	3.769236E-07 1.583721E-04 2.182878E-06	5.356337E-07 1.092634E-04	3.769236E-07 1.078926E-04	5.460217E-05 1.639758E-04	7.702033E-05 1.078926E-04	5.460217E-05 2.182878E-06	1.092634E-04 3.446569E-06
12	2.377352E-07 1.092634E-04 2.165572E-06	3.769237E-07 9.456631E-05	2.978993E-07 5.579538E-05	3.590070E-05 1.078926E-04	5.460217E-05 1.081804E-04	4.111964E-05 1.280996E-06	6.380583E-05 2.182878E-06
13	2.944426E-09 9.567006E-07 2.866753E-06	3.169519E-09 5.759820E-07	1.792340E-09 2.165571E-06	2.978981E-07 2.182877E-06	3.769233E-07 1.280996E-06	2.377350E-07 8.872197E-06	8.513199E-07 5.767948E-06
14	3.169515E-09 1.427202E-06	4.737754E-09 9.567001E-07	3.169515E-09 2.182877E-06	3.769233E-07 3.446568E-06	5.356331E-07 2.182877E-06	3.769233E-07 5.767948E-06	9.567001E-07 1.173895E-06

15	5.767948E-06	3.169519E-09	2.944426E-09	2.377350E-07	3.769233E-07	2.978991E-07	5.758920E-07
	1.793330E-09	8.513199E-07	1.280996E-06	2.182877E-06	2.165571E-06	2.866753E-06	5.767948E-06
	9.567000E-07						
	8.872197E-06						

BELOW IS THE DYNAMIC MATRIX.

1	5.793545E-09 7.730146E-10 1.171052E-12	3.766470E-09 4.652133E-10	1.871990E-09 2.407021E-10	1.749782E-09 3.045545E-10	1.763766E-09 1.920902E-10	1.035045E-09 1.922723E-12	6.878673E-10 2.069706E-12
2	3.766470E-09 1.153180E-09 2.069704E-12	7.665534E-09 7.730144E-10	3.766470E-09 3.045543E-10	1.763766E-09 4.327919E-10	2.784828E-09 3.045543E-10	1.763766E-09 2.069704E-12	7.730144E-10 3.045543E-10
3	1.871990E-09 7.730146E-10 1.922723E-12	3.766470E-09 6.878673E-10	5.793545E-09 1.920902E-10	1.035045E-09 3.045545E-10	1.763766E-09 2.407021E-10	1.749782E-09 1.171052E-12	4.653133E-10 2.069706E-12
4	1.414118E-09 8.828484E-08 1.552410E-10	1.425419E-09 5.155509E-08	8.364904E-10 3.322466E-08	8.740976E-08 4.411854E-08	8.717718E-08 2.900776E-08	4.508265E-08 1.045275E-10	7.640956E-08 2.461310E-10
5	1.425419E-09 1.279647E-07 2.461310E-10	2.250609E-09 8.828484E-08	1.425419E-09 4.411855E-08	8.717718E-08 6.223243E-08	1.324924E-07 4.411855E-08	8.717718E-08 2.461310E-10	8.828484E-08 3.407685E-10
6	8.364904E-10 8.828484E-08 1.945275E-10	1.425419E-09 7.640956E-08	1.414118E-09 2.900776E-08	4.508265E-08 4.411854E-08	8.717718E-08 3.322466E-08	8.740976E-08 1.552410E-10	5.155509E-08 2.461310E-10
7	5.559121E-10 1.307206E-07 3.760511E-10	6.247255E-10 7.371448E-08	3.760511E-10 7.640958E-08	7.640956E-08 8.828485E-08	8.828484E-08 5.155511E-08	5.155510E-08 5.559121E-10	1.201716E-07 6.247254E-10
8	6.247254E-10 1.938861E-07 6.247254E-10	9.319634E-10 1.307206E-07	6.247254E-10 8.828486E-08	8.828484E-08 1.279647E-07	1.279647E-07 8.828486E-08	8.828484E-08 6.247254E-10	1.307206E-07 0.310633E-10
9	3.760511E-10 1.307206E-07 5.559121E-10	6.247255E-10 1.201716E-07	5.559121E-10 5.155511E-08	5.155510E-08 8.828485E-08	8.828484E-08 7.640958E-08	7.640956E-08 3.760511E-10	7.371448E-08 6.247254E-10
10	1.945276E-10 8.828486E-08 8.364906E-10	2.461312E-10 5.155511E-08	1.552411E-10 8.740978E-08	3.322467E-08 8.717721E-08	4.411855E-08 4.508267E-08	2.900776E-08 1.414118E-09	7.640958E-08 1.425419E-09
11	2.461311E-10 1.279647E-07 1.425419E-09	3.457689E-10 8.828486E-08	2.461311E-10 8.717721E-08	4.411855E-08 1.324925E-07	6.223244E-08 8.717721E-08	4.411855E-08 1.425419E-09	8.828486E-08 2.250609E-09
12	1.552411E-10 8.828486E-08 1.414118E-09	2.461312E-10 7.640958E-08	1.945276E-10 4.508267E-08	2.900776E-08 8.717721E-08	4.411855E-08 8.740978E-08	3.322467E-08 8.364906E-10	5.155511E-08 1.425419E-09
13	1.922710E-12 7.730136E-10 1.871990E-09	2.069696E-12 4.653127E-10	1.171045E-12 1.749782E-09	2.407017E-10 1.763765E-09	3.045540E-10 1.035045E-09	1.920899E-10 5.793545E-09	6.878664E-10 3.766470E-09
14	2.065694E-12 1.153179E-09	3.063754E-12 7.730137E-10	2.069694E-12 1.763765E-09	3.045540E-10 2.784827E-09	4.327915E-10 1.763765E-09	3.045540E-10 3.766470E-09	7.730137E-10 7.665535E-09

15	3.766470E-09	1.171045E-12	2.069696E-12	1.922710F-12	1.920999E-10	3.045540E-10	2.407017E-10	4.653127E-10
		7.730136E-10	6.878664E-10	1.035045E-09	1.763765F-09	1.749782E-09	1.871990F-09	2.766470E-09
		5.793545E-09						

BELOW IS THE MASS MATRIX.

1	0.0006530
2	0.0006530
3	0.0006530
4	0.0008080
5	0.0008080
6	0.0008080
7	0.0008080
8	0.0008080
9	0.0008080
10	0.0008080
11	0.0008080
12	0.0008080
13	0.0006530
14	0.0006530
15	0.0006530

1	3.24E-03	-1.82E-10	3.21E-11	-2.77E-12
2	-1.77E-10	3.24E-03	1.32E-11	-2.14E-11
3	3.08E-11	6.48E-12	3.24E-03	2.11E-11
4	-5.16E-12	-1.90E-11	2.31E-11	3.23E-03

ABOVE IS THE MATRIX V TRANSPOSE M V ,WHERE V IS THE MATRIX OF MODE SHAPES.

THE MAXIMUM AND MINIMUM DIAGONAL VALUES ARE 0.003 AND 0.003

THE MAXIMUM OFF DIAGONAL MAGNITUDE = 0.00000000

BELOW IS THE INERTIA MATRIX.

1	0.0032358
2	0.0032256
3	0.0032365
4	0.0032330

BELOW ARE THE SHAPES FOR MODES 1 THROUGH 4

1	0.0064446	1	0.0263932	1	-0.0098159	1	-0.0194774
2	0.0001141	2	0.0373257	2	0.0000000	2	-0.0000000
3	0.0064446	3	0.0263932	3	0.0098159	3	0.0194774
4	0.5011780	4	0.7071068	4	-0.7080313	4	-1.0000000
5	0.7097127	5	1.0000000	5	0.0000000	5	0.0000000
6	0.5011780	6	0.7071067	6	0.7080313	6	1.0000000
7	0.7071067	7	0.0000001	7	-1.0000000	7	-0.0000002
8	1.0000000	8	0.0000001	8	0.0000000	8	-0.0000000
9	0.7071067	9	0.0000000	9	1.0000000	9	0.0000001
10	0.5011781	10	-0.7071067	10	-0.7080315	10	0.9999999
11	0.7087730	11	-0.9999999	11	-0.0000000	11	0.0000000
12	0.5011781	12	-0.7071067	12	0.7080315	12	-0.9999999
13	0.0064446	13	-0.0263932	13	-0.0098159	13	0.0194774
14	0.0091141	14	-0.0373257	14	0.0000000	14	0.0000000
15	0.0064446	15	-0.0263932	15	0.0098159	15	-0.0194774

TABLE OF FINAL RESULTS

MODE NUMBER	RADIANS PER SECOND	FREQUENCY IN CPS	PERCENTAGE ACCURACY OF FREQUENCY	MODE NUMBER
1	1164.7078094	185.3600052	100.000	1
2	2759.8710938	439.2471237	100.000	2
3	3499.4971619	556.9622650	100.000	3
4	5121.7664795	815.1544495	100.000	4

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13. ABSTRACT A user's guide is presented for a computer program developed to aid in the design of sonic-fatigue-resistant aircraft structure. The program employs matrix methods to calculate statistical measurements of response (deflection and stress) for complex structure subjected to pressure loads random in both time and space. The program is in two phases. Finite-element methods are used in the first phase to determine structural characteristics such as flexibility, natural frequencies, and modes of vibration. In the second phase, a cross-power spectral density loading function, is generated and combined with structural characteristics to compute response. Either cross power spectral density or joint statistical moments, including second spectral moments useful in fatigue analysis, can be computed for response. The loading function models a decayed progressive wave typical of laboratory noise sources. Different loading functions can be supplied by the user, because the program is constructed in modular form. The program was written for the IBM 7094 computer primarily in FORTRAN IV language with a MAP language matrix manipulation module.		

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Computer Program User's Guide Matrix Structural Analysis Methods Finite-Element Structural Methods Random-Vibration Analysis Systems Acoustic Vibration Analysis System Structural Vibration Analysis System Sonic Fatigue Analysis Stiffness Matrix Stress Deflection Matrix Eigenvalues Random, Fluctuating Pressure Load Numerical Analysis Method						

Unclassified

Security Classification